

# Improving the Performance of a Pittsburgh Learning Classifier System Using a Default Rule

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**Abstract.** An interesting feature of encoding the individuals of a Pittsburgh learning classifier system as a decision list is the emergent generation of a default rule. However, performance of the system is strongly tied to the learning system choosing the correct class for this default rule. In this paper we experimentally study the use of an explicit (static) default rule. We first test simple policies for setting the class of the default rule, such as the majority/minority class of the problem. Next, we introduce some techniques to automatically determine the most suitable class.

## 1 Introduction

One of the ways to solve classification problems using a genetic algorithm [1, 2] is called Pittsburgh approach [3] or Pittsburgh learning classifier system. The individuals of this system encode a full and variable-length rule set and the solution proposed is the best individual of the population. There are several encoding options for an individual. One of them is coding an individual as a decision list [4] (an ordered set of rules). If we apply this strategy in the evolutionary framework, often the system evolves a default rule. That is, a rule that matches any input instance.

Default rules can be very useful in combination with a decision list because the size of the rule set can be reduced significantly. For instance, for the 11-bit multiplexer we can obtain a rule set of 9 rules instead of 16 unordered ones, as represented in Figure 1. With a smaller rule set, the search space is reduced resulting in two potential advantages: (1) the learner can learn fewer rules faster (representing only the other classes of the dataset) and (2) with a smaller rule

set the system may be less sensitive to over-learning, potentially increasing the test accuracy of the system.

**Fig. 1.** Unordered and ordered rule sets for the MX-11 domain

Unordered MX-11 rule set									
0 0 0 0 # # # # # # # # : 0									
0 0 0 1 # # # # # # # # : 1									
0 0 1 # 0 # # # # # # # # : 0									
0 0 1 # 1 # # # # # # # # : 1									
0 1 0 # # 0 # # # # # # # # : 0									
0 1 0 # # 1 # # # # # # # # : 1									
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1 1 1 # # # # # # # 0 # # # # # # # # : 0									
1 1 1 # # # # # # # 1 # # # # # # # # : 1									

Ordered MX-11 rule set									
0 0 0 0 # # # # # # # # : 0									
0 0 1 # 0 # # # # # # # # : 0									
0 1 0 # # 0 # # # # # # # # : 0									
0 1 1 # # # 0 # # # # # # # # : 0									
1 0 0 # # # # 0 # # # # # # # # : 0									
1 0 1 # # # # # 0 # # # # # # # # : 0									
1 1 0 # # # # # # 0 # # # # # # # # : 0									
1 1 1 # # # # # # # 0 # # # # # # # # : 0									
# # # # # # # # # # # # # # : 1									

The objective of this paper is to investigate the potential benefits of using an explicit and static default rule in a Pitt LCS. Along those lines, the question arises which is the best default class to use. Simple strategies may use the majority class. However, our tests show that dependent on the problem, the minority class may be better as the default class choice. Thus, we develop a mechanism that is able to automatically determine the best class for the default rule.

The rest of the paper is structured as follows: Section 2 shows some related work. Next, Section 3 describes briefly the main characteristics of the system used in this paper. Later, Section 4 illustrates the motivation of using a default rule, followed by Section 5 that reports the modifications applied to the knowledge representation of the system to integrate the default rule. Next, Section 6 shows

some illustrative results of the simple policies for the default rule. After the simple policies, we describe the more sophisticated ones in Section 7. Section 8 shows the experimentation results of applying the described policies. Finally, Section 9 presents conclusions and further work.

## 2 Related Work

We can find previous uses of a static default rule in the *LCS* field, although not in an explicit way: Classic Pitt-approach systems such as *GABIL* [3] or *GIL* [5], which perform concept learning (learning a concept from sets of positive/negative examples), implicitly have a default rule that covers the negative examples. The rules generated do not have an associated class because all of them cover the positive examples. However, there is no explicit policy to decide which set is the positive or negative one in order to learn better. The decision simply comes from the definition of the dataset.

Looking at the machine learning field in general we find other examples of default rules. The C4.5 rule system [6] uses an explicit default rule and, alike our system, it generates a rule set acting as a decision list. To select the class for this default rule, it uses the class that has less instances covered by the other rules in the rule set. This kind of approach seems feasible when we have induced the rule set beforehand, instead of using it during learning as our system does.

The *IREP* system [7] induces the rules in order, modeling each class of the problem (using the instances of the classes still to be learned as negative examples). The criteria of this global order is ascendant frequency of examples. Therefore, the default rule of this system uses a majority class policy.

## 3 Framework

GAssist [8] is a Pittsburgh genetic-based machine learning system descendant of *GABIL* [3]. The system applies a near-standard *GA* that evolves individuals that represent complete problem solutions. An individual consists of an ordered, variable-length rule set. Directly from *GABIL* we have taken the semantically correct crossover operator for variable-length individuals.

Dealing with variable-length individuals raises some important issues. One of the most important one is the control of the size of the evolving individuals [9]. This control is achieved in GAssist using two different operators:

1. *Rule deletion*. This operator deletes the rules of the individuals that do not match any training example. This rule deletion is done after the fitness computation and has two constraints:
  - (a) The process is only activated after a predefined number of iterations (to prevent an irreversible diversity loss)
  - (b) The number of rules of an individual never decreases below a threshold. This introduces some “neutral code” that can protect the individuals from the disruptive effect of the crossover operator.

2. *Minimum description length-based fitness function.* The minimum description length (*MDL*) principle [10] is a metric applied in general to a theory (being a rule set in this paper) which balances the complexity and accuracy of the rule set. In previous work we developed a fitness function based on this principle. A detailed explanation of the fitness function can be found in [11].

The knowledge representation used for real-valued attributes is called *adaptive discretization intervals* rule representation (*ADI*) [12]. This representation uses the semantics of the *GABIL* rules (conjunctive normal form predicates), but applies non-static intervals formed by joining several neighbor discretization intervals. These intervals can evolve through the learning process splitting or merging among them potentially using several discretizers at the same time.

Parameters of the system are set as follows: Crossover probability 0.6; tournament selection; tournament size 3; population size 300; Individual-wise mutation probability 0.6; initial number of rules per individual 20; probability of “1” in initialization 0.75; Rule Deletion Operator: Iteration of activation: 5; minimum number of rules: number of active rules +3; MDL-based fitness function: Iteration of activation 25; initial theory length ratio: 0.075; weight relax factor: 0.9. ADI knowledge representation: split and merge probability: 0.05; reinitialize probability at initial iteration: 0.02; reinitialize probability at final iteration: 0; merge restriction probability: 0.5; maximum number of intervals: 5; set of uniform discretizers used: 4, 5, 6, 7, 8, 10, 15, 20 and 25 bins; iterations: maximum of 1500.

## 4 Motivation

In order to illustrate the benefits of the default rule, we show the results of running the system with no static default rule for the *Glass* problem from the *UCI* repository [13] in table 1. We used stratified ten-fold cross validation for the tests and a hundred random seeds for each fold (a total of 1000 runs, unlike the 15 seeds and 150 runs used in the rest of the paper).

We can see the benefits of using a default rule and, more importantly, the benefits of choosing the correct class for the default rule. The choice of the class for the default rule has a significant influence on the resulting accuracy, suggesting that a good default rule choice can improve learning performance and generality of the resulting solution.

## 5 Static Default Rule Mechanism

To force the usage of a default rule, few modifications are necessary: we only need to codify our individuals as decision lists, independent of the knowledge representation used. The implementation of the static default rule is very simple. Basically it affects only the matching function classifying any input instance by the default class if no rule (in the decision list) matches the instance. The

**Table 1.** How the generation of a default rule can affect the performance in the *Glass* dataset

Runs generating a default rule	736
Runs not generating a default rule	264
Accuracy of runs with a default rule	66.98±8.00
Accuracy of runs without a default rule	66.27±7.79
Average accuracy of runs using class 1 as default rule	65.45±7.39
Average accuracy of runs using class 2 as default rule	67.76±7.81
Average accuracy of runs using class 3 as default rule	59.40±5.51
Average accuracy of runs using class 4 as default rule	66.18±8.70
Average accuracy of runs using class 5 as default rule	67.66±8.58
Average accuracy of runs using class 6 as default rule	64.48±7.36

pseudocode in Figure 2 clarifies this mechanism. Additionally, the default rule class is removed from the classes that can be used by the rest of the rules in the population, effectively reducing the search space. A general representation of the extended rule set is shown in Figure 3.

1. We determine with some criterion (in the following sections several criteria are studied) which class is the default class.
2. An individual predicts this default class when no rule matches an input instance.
3. The other rules of the individual cannot use the default class. Neither initialization nor mutation can make a regular rule of the individual point to the default class.
4. The default rule is included in the size of the rule set. This means that the rest of the system transparently sees an individual with one more rule. This affects the parts of the fitness formula that uses the size of the rule set as a variable.
5. The default rule cannot be affected by crossover, mutation nor any other recombination operator.
6. The rule deletion operator ignores the petitions to delete this rule, in the rare chance that this rule matches nothing (all problem instances are covered by other rules already).
7. The MDL-based fitness function computes a theory length for this rule supposing that the rule is totally general, that is, as if it were the emergent default rule observed before implementing this mechanism.

For the specific case of two-class domains, the classification problem is transformed into a concept learning problem and the resulting knowledge representation is quite close to the ones used in other evolutionary concept learning systems like *GABIL* [3] or *GIL* [5].

## 6 Simple Policies Determining the Default Rule Class

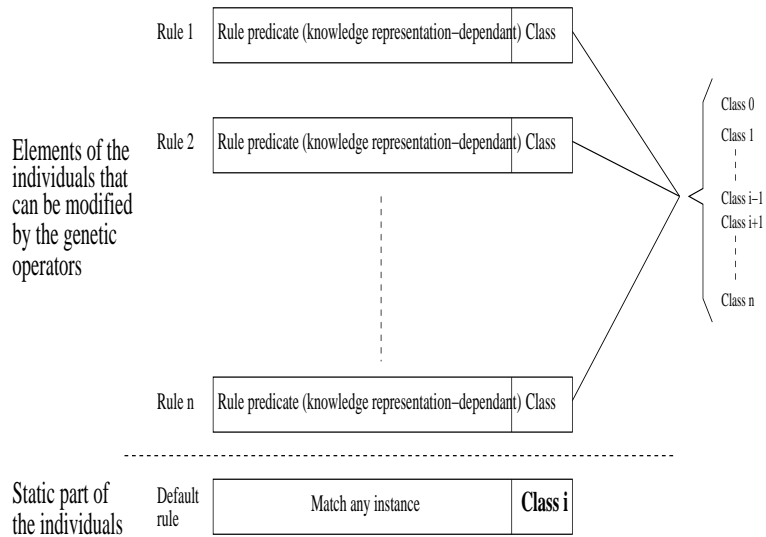
In order to answer the question of which class is suitable for being the default class we start by experimenting with two simple policies: using the most and

**Fig. 2.** Match process using an static default rule

```

Match process
Input : RuleSet, Instance
Index = 0
Found = false
While Index < RuleSet.size and not Found Do
  If RuleSet.rule[Index] matches Instance Then
    Class = RuleSet.rule[Index].class
    Found = true
  Else
    Index ++
  EndIf
EndWhile
If not Found Then
  Class = DefaultClass
EndIf
Output : Predict class Class for instance Instance
  
```

**Fig. 3.** Representation of the extended rule set with the static default rule



least frequent class in the domain. In Section 8 we can see the results of these tests for several datasets. Here we show the results (in Table 2) of only two datasets (*Glass* and *Ionosphere*), also from UCI. For *Glass* the best policy is using the majority class. For *Ionosphere* the best policy is using the minority class. The point of showing these two datasets is that it is very difficult to decide *a priori* which is the most suitable default rule class for each dataset. The values of the train accuracy and the number of rules give hints about how to combine the two policies to maximize the performance of the system. In Section 8 we show a simple combination consisting of choosing at the test stage the policy which has more train accuracy.

**Table 2.** Results using majority and minority policy for the default class in the *Glass* and *Ionosphere* datasets.

Domain	Def. Class.	Policy	Train accuracy	Test accuracy	Number of rules
Glass	disabled		79.9±2.6	66.4±8.1	6.4±0.7
Glass	majority		83.2±1.6	69.5±6.9	6.6±0.8
Glass	minority		80.6±2.3	66.7±8.0	7.2±0.8
Ionosphere	disabled		96.0±0.6	92.8±3.6	2.3±0.6
Ionosphere	majority		95.7±0.8	90.0±4.4	5.7±1.2
Ionosphere	minority		96.8±0.7	93.0±3.7	2.6±0.8

## 7 Automatically Determined Default Rule Class

Given that the majority class does not always suite best as default class, the next step is to modify the system to automatically determine the best default class. Our initial approach simply assigns a randomly chosen class as default class to each individual in the initial population. Additionally, we introduce a restricted mating mechanism to avoid crossover operations between individuals having different default classes, summarized by the code in Figure 4. Having removed the default class from the rest of the rules, crossing individuals with different default classes may create lethals with high probability. Especially in the specific case of two-class domains, the regular rules of individuals using different default classes cover completely different subsets of rules. Therefore, it is impossible to integrate the rules of these two individuals using the regular crossover operator.

If we run the system in this setting, we observed that usually all individuals with one default class take over the population. The question is if the system is able to choose the correct default class during the initial iterations. To answer this question, we show the evolution of the train accuracy and the number of rules for the *Ionosphere* tests described in the previous section in Figure 5. We can see that the train accuracy of the default class policy using the suitable class for this problem (that is, the minority class) is lower at the initial iterations than the accuracy of the majority class policy. Also, we can see the reason for the

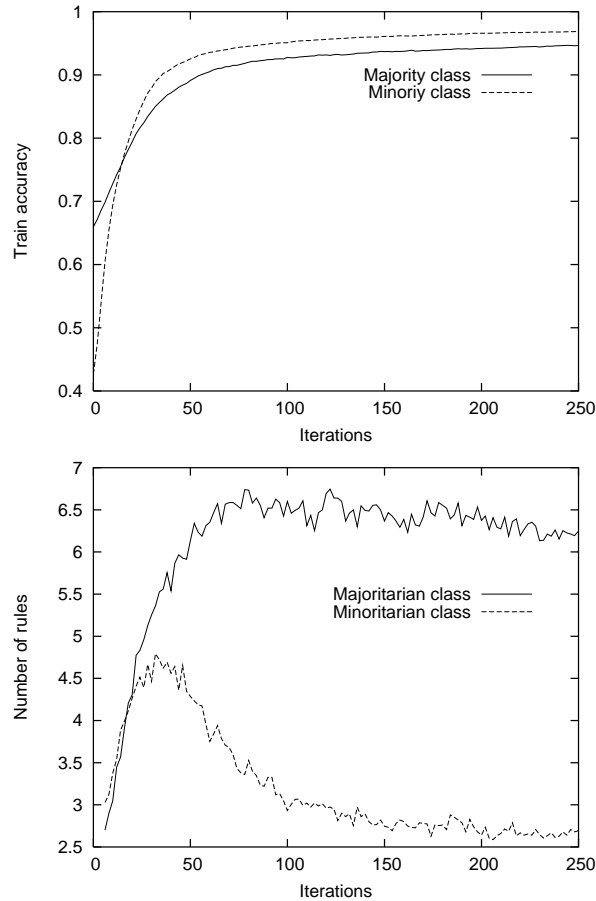
**Fig. 4.** Code of the crossover algorithm with restricted mating

```
Niched crossover algorithm  
Comment To simplify the code, Parents contains only the parent individuals  
Comment already selected for crossover by the probability of crossover  
Input : Parents  
OffspringSet =  $\emptyset$   
While Parents is not empty  
    Parent1 = select randomly and individual from Parents  
    Remove Parent1 from Parents  
    Niche = default class of Parent1  
    If there are individuals in Parents belonging to Niche  
        Parent2 = select randomly and individual from Parents  
        belonging to Niche  
        Remove Parent2 from Parents  
        Offspring1, Offspring2 = apply crossover to Parent1, Parent2  
        Add Offspring1, Offspring2 to OffspringSet  
    Else  
        Offspring = clone of Parent1  
        Add Offspring to OffspringSet  
    EndIf  
EndWhile  
Output : OffspringSet
```



better test accuracy of the minority policy in the smaller (better generalized) rule set created by this policy.

**Fig. 5.** Evolution of the train accuracy and the number of rules for the Ionosphere problem using majority/minority default class policies



Thus, it appears necessary to introduce an additional niching mechanism that preserves individuals for all default classes until the system has learned enough to decide correctly on the best default class. This niching is achieved using a modified tournament selection mechanism, inspired by [14] in which the individuals participating in each tournament are forced to belong to the same class. Also, each default class has an equal number of tournaments. This niched tournament selection is represented by the pseudocode in Figure 6. The tournament with niche preservation is used until the best individuals of each

default class have similar train accuracy. After this point, the niching is disabled and the system chooses freely among the individuals. Specifically, we compute for each niche the average accuracy over the last 15 iterations of its best individual. When the standard deviation of all these averages is smaller than 0.5%, we disable the niched tournament selection, effectively enabling the superior default class to take over the whole population.

**Fig. 6.** Pseudocode for the niched tournament selection

```

Niched tournament selection
Input : Population, PopSize, NumNiches, TournamentSize
NextPopulation =  $\emptyset$ 
For  $i = 1$  to NumNiches
    ProportionNiche[ $i$ ] = PopSize/NumNiches
EndFor

For  $i = 1$  to PopSize
    Niche = select randomly a niche based on ProportionNiche
    ProportionNiche[Niche] --
    Select TournamentSize individuals from Population belonging to Niche
    winner=Apply tournament
    Add winner to NextPopulation
EndFor
Output : NextPopulation

```

To summarize, the changes introduced to the default rule model by the automatic policy are the following:

1. Initialization assigns randomly to each individual a class as being the default class.
2. This class cannot be used in the regular rules of the individual.
3. Individuals having different default classes cannot exchange rules. The crossover algorithm is modified adding this mating restriction.
4. Niched tournament selection preserves an uniform proportion of individuals from all default classes in the population. This niching process is achieved reserving a quota of tournaments to each niche and only applying tournaments among individuals belonging to the same niche.
5. The niching mechanism is disabled when individuals using different default classes can compete fairly among themselves. Specifically, we compute, for each default class, the average accuracy over the last 15 iterations of its best individual. When the standard deviation of all these averages is smaller than 0.5%, the niched tournament selection is disabled and a regular tournament selection takes places until the end of the learning process.

## 8 Results

In this section, we show the results of comparing the three policies tested for the default class (*majority, minority, auto* to the original system (*orig*) with emergent

default rule. The tests include 15 datasets used previously in [12], summarized in table 3. Each dataset has been partitioned into training/test sets using stratified ten-fold cross-validation [15], and having for each fold the tests repeated 15 times.

**Table 3.** Features of the datasets used in the experimentation of this paper

Dataset Properties								
Domain	#Inst.	#Attr.	#Real	#Nom.	#Cla.	Dev.cla.	Maj.cla.	Min.cla.
bpa	345	6	6	—	2	7.97%	57.97%	42.03%
bps	1027	24	24	—	2	1.60%	51.61%	48.39%
bre	699	9	9	—	2	15.52%	65.52%	34.48%
gls	214	9	9	—	6	12.69%	35.51%	4.21%
h-s	270	13	13	—	2	5.56%	55.56%	44.44%
ion	351	34	34	—	2	14.10%	64.10%	35.90%
lrn	648	6	4	2	5	14.90%	45.83%	1.54%
mmg	216	21	21	—	2	6.01%	56.02%	43.98%
pim	768	8	8	—	2	15.10%	65.10%	34.90%
son	208	60	60	—	2	3.37%	53.37%	46.63%
thy	215	5	5	—	3	25.78%	69.77%	13.95%
veh	846	18	18	—	4	0.89%	25.77%	23.52%
wdbc	569	30	30	—	2	12.74%	62.74%	37.26%
wine	178	13	13	—	3	5.28%	39.89%	26.97%
wdbc	198	33	33	—	2	26.26%	76.26%	23.74%

Table 4 shows the results for these tests, also including a fifth configuration (*majority+minority*), in which the majority/minority policy is chosen in the test stage that obtained more training accuracy. This configuration usually chooses the correct policy (although there are some exceptions, like *bpa*). The results were analyzed using pair-wise statistical t-tests with Bonferroni correction to determine how many times each method could significantly outperform or be outperformed by the other methods. These statistical tests are summarized in table 5.

At first glance, we can see that all but two datasets (*wbcd* and *wdbc*) can benefit (by one or more of the studied default class policies) from the inclusion of a default rule. However, the achieved accuracy increase is not uniform across the datasets. Some of them, like *gls* or *son*, show a notable accuracy increase, while some others only show a small, non-significant increase. To understand these different degrees of accuracy increase we have computed the percentage of runs where the *orig* configuration was already generating a default rule emergently. Table 6 shows these results including the accuracy of the *orig* configuration as well as the accuracy of the best default class policy for each dataset (and their difference). Although it is not totally clear, we can see a correlation between the percentage of discovered default rules and the accuracy difference between using/not using the default rule. The clearest exception is the *gls* dataset. However, considering that this dataset has 6 classes, the benefits of removing the default class from the pool of classes used in the regular rules are already substantial even if the *orig* configuration was already using a default rule.

**Table 4.** Results of the tests comparing the studied default class policies to the original configuration using pop. size 300

Domain	Result	Default rule policy				
		Disabled	Major	Minor	Auto	Major+Minor
bpa	Train	78.6±1.6	81.4±1.3	80.1±1.6	80.8±1.4	81.4±1.3
	Test	63.8±7.4	62.9±7.8	65.2±6.5	64.0±6.9	62.9±7.8
	#rules	6.7±1.0	8.9±1.4	8.3±1.5	8.5±1.6	8.9±1.4
bps	Train	84.8±0.9	86.0±0.7	86.8±0.7	86.6±0.7	86.8±0.7
	Test	80.1±3.9	81.2±3.6	81.5±3.6	81.4±3.7	81.5±3.6
	#rules	5.1±0.4	6.1±1.1	5.7±0.9	5.6±0.8	5.7±0.9
bre	Train	97.7±0.3	98.2±0.3	98.4±0.3	98.4±0.3	98.4±0.3
	Test	95.9±2.2	95.0±2.5	95.7±2.0	95.6±2.2	95.7±2.0
	#rules	2.6±0.7	5.8±1.2	3.2±0.6	3.3±0.7	3.2±0.6
gls	Train	79.9±2.6	83.2±1.6	80.6±2.3	79.0±1.8	83.2±1.6
	Test	66.4±8.1	69.5±6.9	66.7±8.0	66.9±7.4	69.5±6.9
	#rules	6.4±0.7	6.6±0.8	7.2±0.8	6.9±0.9	6.6±0.8
h-s	Train	89.8±1.2	91.6±0.9	92.1±0.8	91.9±0.9	92.1±0.8
	Test	79.5±6.2	79.3±6.4	81.3±6.8	81.3±6.1	81.3±6.8
	#rules	6.7±0.9	7.6±1.2	7.3±1.2	7.4±1.3	7.3±1.2
ion	Train	96.0±0.6	95.7±0.8	96.8±0.7	96.8±0.7	96.8±0.7
	Test	92.8±3.6	90.0±4.4	93.0±3.7	93.1±3.9	93.0±3.7
	#rules	2.3±0.6	5.7±1.2	2.6±0.8	2.6±0.7	2.6±0.8
lrn	Train	75.2±1.9	76.8±0.8	75.4±1.4	75.4±1.0	76.8±0.8
	Test	68.5±4.7	68.9±5.7	68.9±4.5	68.6±5.6	68.9±5.7
	#rules	8.5±1.9	9.6±1.9	9.2±1.9	8.6±1.7	9.6±1.9
mmg	Train	79.7±1.8	83.2±1.3	83.1±1.3	83.0±1.4	83.2±1.3
	Test	66.2±7.8	68.9±8.3	67.8±8.4	66.8±9.0	68.9±8.3
	#rules	6.5±0.8	6.7±0.9	6.7±0.8	6.6±0.9	6.7±0.9
pim	Train	79.7±0.9	81.3±0.8	80.9±0.7	81.1±0.8	81.3±0.8
	Test	74.7±4.7	75.4±4.8	75.0±4.7	75.0±4.5	75.4±4.8
	#rules	5.2±0.4	6.2±1.0	5.6±0.8	6.1±1.0	6.2±1.0
son	Train	92.2±1.6	96.1±1.2	94.8±1.4	95.5±1.4	96.1±1.2
	Test	72.6±11.5	77.0±9.0	76.1±9.7	76.1±9.3	77.0±9.0
	#rules	6.7±1.1	7.6±1.4	7.7±1.3	7.4±1.1	7.6±1.4
thy	Train	97.4±1.0	98.4±0.7	98.4±0.7	98.1±0.8	98.4±0.7
	Test	91.9±5.6	92.8±4.8	92.3±5.3	92.2±5.6	92.8±4.8
	#rules	5.2±0.4	5.7±0.6	5.4±0.5	5.5±0.6	5.7±0.6
veh	Train	71.1±2.2	73.5±1.4	73.5±1.4	72.0±1.5	73.5±1.4
	Test	66.4±4.7	68.1±4.5	67.4±4.9	67.5±4.7	68.1±4.5
	#rules	6.6±1.2	9.3±2.0	9.9±1.6	8.0±1.8	9.3±2.0
wdbc	Train	97.2±0.8	97.8±0.6	97.8±0.6	97.8±0.7	97.8±0.6
	Test	94.1±3.0	94.2±3.1	94.0±3.0	94.3±3.1	94.2±3.1
	#rules	4.3±1.1	4.6±0.9	4.4±1.0	4.5±1.0	4.6±0.9
wine	Train	99.4±0.5	99.7±0.4	99.9±0.3	99.6±0.4	99.9±0.3
	Test	92.7±5.9	93.3±6.2	92.2±6.3	93.9±5.9	92.2±6.3
	#rules	3.8±0.7	3.6±0.6	4.1±0.5	3.8±0.6	4.1±0.5
wdbc	Train	84.3±3.0	89.4±2.0	86.4±3.4	88.7±2.3	89.4±2.0
	Test	76.0±7.3	75.8±7.4	72.6±8.5	75.2±7.5	75.8±7.4
	#rules	2.8±0.8	3.8±0.9	4.2±1.2	3.6±1.0	3.8±0.9
ave.	Train	86.9±9.0	88.8±8.4	88.3±8.8	88.3±9.0	89.0±8.5
	Test	78.8±11.4	79.5±10.7	79.3±11.0	79.5±11.3	79.8±10.9
	#rules	5.3±1.8	6.5±1.8	6.1±2.1	5.9±1.9	6.1±2.1

**Table 5.** Summary of the statistical t-tests applied to the experimentation results of popsize 300, with a confidence level of 0.05. Cells in table count how many times the method in the row significantly outperforms the method in the column.

Policy	Disabled	Major	Minor	Auto	Major+Minor	Total
Disabled	-	2	1	0	0	3
Major	3	-	2	1	0	6
Minor	2	2	-	0	0	4
Auto	2	1	1	-	0	4
Major+Minor	4	2	2	1	-	9
Total	11	7	6	2	0	

**Table 6.** Percentage of runs where *orig* configuration was already generating a default rule, accuracy difference between *orig* and the best default class policy for each dataset.

Rows are sorted by the percentage of default rule generation in *orig*

Label	meaning			
DRG	Percentage of runs where the default rule was generated in <i>orig</i> configuration			
AccO	Accuracy of the <i>orig</i> configuration			
AccDR	Accuracy of the best rule policy on the dataset			
AccDif	Accuracy difference between AccO and AccDR			
Dataset	DRG	AccO	AccDR	AccDif
mmg	19.33%	66.21%	68.88%	-2.67%
son	36.00%	72.58%	76.99%	-4.42%
bps	40.00%	80.10%	81.55%	-1.44%
veh	46.67%	66.43%	68.15%	-1.72%
pim	50.67%	74.65%	75.37%	-0.71%
wdbc	55.33%	94.06%	94.26%	-0.20%
h-s	57.33%	79.46%	81.31%	-1.85%
bpa	65.33%	63.79%	65.22%	-1.43%
thy	68.67%	91.92%	92.79%	-0.87%
wine	71.33%	92.74%	93.85%	-1.12%
glc	74.00%	66.37%	69.52%	-3.15%
lrn	76.00%	68.55%	68.93%	-0.39%
wdbc	82.00%	76.03%	75.78%	0.25%
ion	86.00%	92.85%	93.13%	-0.29%
bre	96.00%	95.88%	95.74%	0.14%

From the test accuracy averages and the t-test results it is clear that the *major+minor* policy is the best configuration, both in performance and robustness, because it has been never outperformed in a significant way. However, having in this configuration a run-time two times larger than in the other configurations, we have to question whether the computational cost sacrifice is worth it. Looking at the other configurations, *major* and *auto* are tied in accuracy average, but *auto* is much more robust than *major* according to the t-tests.

Nevertheless, it is important to investigate why the *auto* policy reaches a lower performance than *major+minor*. Table 7 shows the class distribution of the default rules that appear in the *auto* configuration runs. We can see that this configuration is not able to determine, which is the most suitable default class. Actually, on only 5 of the 15 datasets the chosen default class was almost or totally concentrated on a single class.

**Table 7.** Default class behavior in the auto configuration

Dataset	Major. class pos.	Minor. class pos.	Class distribution in default rule
bpa	2	1	(50.67%,49.33%)
bps	1	2	(14.67%,85.33%)
bre	1	2	(0.00%,100.00%)
gls	2	4	(14.00%,40.00%,8.67%,9.33%,14.00%,14.00%)
h-s	1	2	(32.00%,68.00%)
ion	2	1	(97.33%,2.67%)
lrn	1	5	(17.33%,35.33%,34.00%,11.33%,2.00%)
mmg	1	2	(48.00%,52.00%)
pim	1	2	(62.00%,38.00%)
son	2	1	(32.00%,68.00%)
thy	1	3	(40.67%,18.67%,40.67%)
veh	3	4	(35.33%,24.00%,13.33%,27.33%)
wdbc	2	1	(48.00%,52.00%)
wine	2	3	(4.00%,70.67%,25.33%)
wpbc	2	1	(1.33%,98.67%)

Another important issue is the number of iterations where the niched tournament selection was used. Table 8 shows these results. We can see that for some datasets, the niching process was used for quite a long time.

It is reported in the niching literature [16] that we should increase the population size in order to guarantee that all niches can be learned properly. For this reason, a second set of tests was performed increasing the population size from 300 to 400. The results are shown in table 9. The summary of the statistical t-tests applied to these results is in table 10.

Now we can see a different picture. The increase in population size actually enables the *auto* policy to permit all niches to be learned properly. This fact is reflected by the accuracy performance of this policy, which manages to reach *major+minor*, both in accuracy and in robustness, based on the t-tests. Now that both policies are competitive, the smaller computational cost of *auto* (also compared to *major+minor* using a population size of 300) clearly makes it the most suitable configuration for the default class.

**Table 8.** Percentage of iterations that used the niched tournament selection in the default rule auto configuration

Dataset	Percentage of iterations
bpa	8.19%
bps	15.10%
bre	13.71%
gls	27.82%
h-s	13.33%
ion	6.72%
lrn	69.06%
mmg	10.79%
pim	9.41%
son	15.45%
thy	30.20%
veh	20.29%
wdbc	7.66%
wine	34.11%
wpbc	12.43%

Moreover, we can see how the only method that degrades performance when we increase the population size is the majority class policy, suggesting that the system is sensitive to over-learning in domains where the majority class policy is not suitable. The larger average number of rules and the better training accuracy of the solutions generated by this policy confirm the over-learning problem.

## 9 Conclusions and Future Work

In this paper we have tested some methods that extend the rule-based and decision-list-style knowledge representations for a Pittsburgh Learning Classifier System by using a static default rule. This kind of systems tend to generate an emergent default rule, which can increase the performance of the system. By forcing the representation of a default rule, we intended to guarantee these positive effects.

Simple policies such as using the majority/minority class as the default class perform quite well compared to the original system. However, they perform poorly on certain datasets somewhat showing a lack of robustness. We can almost integrate the best results of both policies by using the simple heuristic of selecting the policy with more training accuracy. This mechanism introduces a good performance boost, but doubles the run-time.

For this reason, we have developed a mechanism that decides automatically the class for the default rule. This technique works by integrating in a single population individuals using all possible default classes and letting them compete among themselves. This approach has a problem, however, which is providing a fair competition framework, because each default rule class can yield different

**Table 9.** Results of the tests comparing the studied default class policies to the original configuration using pop. size 400

Domain	Result	Default rule policy				
		Disabled	Major	Minor	Auto	Major+Minor
bpa	Train	79.3±1.7	82.0±1.4	80.7±1.4	81.0±1.6	82.0±1.4
	Test	64.0±7.5	62.6±7.5	64.4±6.9	64.5±7.3	62.6±7.5
	#rules	6.8±1.0	8.9±1.4	8.3±1.6	8.7±1.4	8.9±1.4
bps	Train	84.9±0.9	86.2±0.7	87.1±0.6	86.9±0.8	87.1±0.6
	Test	80.4±4.5	80.9±3.8	81.6±3.8	81.2±3.9	81.6±3.8
	#rules	5.1±0.4	6.1±1.1	5.9±1.0	5.8±1.0	5.9±1.0
bre	Train	97.7±0.4	98.3±0.3	98.5±0.4	98.4±0.4	98.5±0.4
	Test	95.7±2.3	95.0±2.6	95.7±1.9	95.8±1.9	95.7±1.9
	#rules	2.6±0.8	5.8±1.1	3.3±0.7	3.2±0.7	3.3±0.7
gls	Train	80.8±2.5	83.8±1.6	81.3±2.1	79.5±1.7	83.8±1.6
	Test	66.8±7.0	69.1±7.7	68.0±8.3	67.1±7.4	69.1±7.7
	#rules	6.5±0.7	6.8±0.8	7.5±0.9	6.7±0.8	6.8±0.8
h-s	Train	90.1±1.0	92.0±0.9	92.4±0.8	92.2±0.8	92.4±0.8
	Test	79.4±7.0	79.2±5.8	81.6±6.9	81.2±6.6	81.6±6.9
	#rules	6.6±0.8	7.8±1.3	7.4±1.2	7.4±1.2	7.4±1.2
ion	Train	96.1±0.6	95.9±0.8	97.1±0.7	96.9±0.7	97.1±0.7
	Test	93.5±3.5	90.4±4.3	93.4±3.5	92.8±4.0	93.4±3.5
	#rules	2.3±0.7	5.7±1.2	2.6±0.7	2.6±0.9	2.6±0.7
lrn	Train	75.7±1.7	77.2±0.8	75.8±1.4	75.7±1.0	77.2±0.8
	Test	68.0±5.0	69.1±5.4	68.7±5.2	69.1±4.9	69.1±5.4
	#rules	8.4±1.9	9.5±1.6	9.3±1.9	8.8±1.8	9.5±1.6
mmg	Train	80.3±1.7	83.4±1.3	83.4±1.3	83.5±1.1	83.4±1.3
	Test	65.9±8.3	69.0±8.0	67.3±8.9	69.7±7.7	69.0±8.0
	#rules	6.5±0.8	6.5±0.9	6.8±1.0	6.6±0.9	6.5±0.9
pim	Train	80.0±1.0	81.5±0.7	81.2±0.7	81.4±0.7	81.5±0.7
	Test	74.7±4.6	75.2±4.4	74.8±4.7	74.9±4.6	75.2±4.4
	#rules	5.3±0.6	6.3±1.1	5.8±0.9	6.1±1.0	6.3±1.1
son	Train	92.7±1.5	96.7±1.1	95.3±1.3	96.1±1.3	96.7±1.1
	Test	71.3±9.4	76.2±9.1	74.6±10.1	76.3±8.9	76.2±9.1
	#rules	6.7±1.0	7.6±1.3	7.7±1.5	7.6±1.4	7.6±1.3
thy	Train	97.6±0.9	98.6±0.7	98.6±0.7	98.3±0.8	98.6±0.7
	Test	91.5±6.2	92.0±5.2	92.4±4.8	91.4±5.6	92.4±4.8
	#rules	5.2±0.5	5.7±0.7	5.4±0.6	5.5±0.6	5.4±0.6
veh	Train	71.9±1.9	74.1±1.3	74.2±1.2	72.6±1.3	74.2±1.2
	Test	66.9±4.3	67.6±4.2	68.3±4.5	67.9±4.8	68.3±4.5
	#rules	6.5±1.3	9.4±1.8	10.0±1.8	8.4±1.8	10.0±1.8
wdbc	Train	97.2±0.8	98.0±0.5	97.9±0.6	97.8±0.6	98.0±0.5
	Test	93.9±2.9	94.4±3.1	94.4±3.2	94.4±3.1	94.4±3.1
	#rules	4.3±1.2	4.8±1.1	4.2±0.7	4.5±0.9	4.8±1.1
wine	Train	99.4±0.6	99.7±0.4	99.8±0.3	99.6±0.4	99.8±0.3
	Test	94.1±6.0	93.2±6.4	92.0±6.5	93.2±6.3	92.0±6.5
	#rules	3.8±0.7	3.7±0.6	4.2±0.5	3.8±0.7	4.2±0.5
wdbc	Train	84.9±2.8	89.9±1.8	87.1±3.3	89.0±2.1	89.9±1.8
	Test	76.6±6.7	75.3±7.0	72.4±9.1	76.3±7.1	75.3±7.0
	#rules	2.8±0.9	3.9±0.9	4.4±1.2	3.7±1.0	3.9±0.9
ave	Train	87.2±8.8	89.2±8.3	88.7±8.6	88.6±8.9	89.3±8.3
	Test	78.8±11.5	79.3±10.7	79.3±11.1	79.7±10.8	79.7±11.7
	#rules	5.3±1.7	6.6±1.7	6.2±2.1	6.0±2.0	6.2±2.2



**Table 10.** Summary of the statistical t-tests applied to the experimentation results of popsize 400, with a confidence level of 0.05. Cells in table count how many times the method in the row significantly outperforms the method in the column.

Policy	Disabled	Major	Minor	Auto	Major+Minor	Total
Disabled	-	2	1	0	0	3
Major	1	-	1	0	0	2
Minor	1	3	-	0	0	4
Auto	1	3	1	-	0	5
Major+Minor	2	3	1	0	-	6
Total	5	11	4	0	0	

learning progress. In order to achieve this fairness, we use a niched tournament selection that guarantees that all niches (different default rules) survive in the population until they can compete successfully by themselves. This automatic mechanism performs best when we increase the population size, which is an usual requirement in most systems that use niching, because we have to guarantee that each niche has enough individuals to ensure sufficient diversity for building block supply and thus successful and reliable learning.

The increase in population size for the majority/minority policies, however, showed no performance increase or even some performance decrease, suggesting the amplification of the policy weaknesses. These weaknesses are derived from overlearning, which is reflected in the larger training accuracy and larger average rule set sizes and also on the statistical tests.

Although the automatic policy does not outperform the major+minor policy, the accuracy difference is quite small in most datasets and the computational cost is significantly lower. Therefore, it appears that in most situations the automatic policy is the best method.

One of the main sacrifices done in the *auto* default class determination policy is the mating restriction introduced in the crossover algorithm, preventing the creation of lethals, because it is almost impossible to create competitive offspring if the parents cover different subsets of the training instances. However, it would be useful to study if there are any feasible ways to recombine successfully individuals with different default classes. If we achieve this objective, perhaps we can reduce the population size requirements of the *auto* policy.

Another alternative would be to develop more sophisticated heuristics that combine the simple default class policies. It might be possible to have a method that only requires a short run to reliably decide on the most suitable default rule class, instead of running a full test for each candidate class. To do so, it appears necessary to also investigate in general in which cases which default rule class is most appropriate. It is expected that the best default rule class does not only depend on the class distribution and class boundaries but also, mutually, on the representation of the class boundaries in the evolving rules. Future research will shine further light on this matter.

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