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A FORMAL DEFINITION OF ALGOL 60

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ABSTRACT

This paper contains a formal definition of the semantics of the ECMA subset of Algol 60. The definition is written as a series of recursive functions. Although some of the ideas of the "Vienna Definition Language" are adopted, the use of the control mechanism is specifically avoided.

CONTENTS

1. BACKGROUND

This paper contains a formal definition of the ECMA subset of Algol 60. The definition is written as a family of recursive functions. Specific details of the notation used are given in Chapter 3 below. Before coming to this it is, perhaps, worth reviewing the motivation for this work.

First it must be clear that the hope is to illustrate language definition ideas rather than to provide a definition of an already widely understood language.

The authors were lead to undertake this definition as a result of an interest in correctness proofs for compilers. Some of the work on this topic (begun Ref. 8, see also ref. 9) has been based on "Vienna-style" definitions (generally referred to as VDL definitions). In spite of the relative success of this line of work, it was clear that certain difficulties were the result of the definition method. In particular the way in which the sequencing of operations was handled by the control complicates reasoning about the order of operations. This and other problems are discussed in the next Chapter. Ideas existed for circumventing some of these problems (e.g. ref. 7). The definition given below shows how these ideas are worked out in detail on a reasonable size language.

There is, of course, an existing formal definition of Algol 60 in the Vienna-style (ref. 6). The reader will find that differences exist between that and the current definition other than those caused by the discussed changes of method. Most of these can be explained by an attempt to come closer to the present authors' understanding of the "spirit of Algol". The best example of this is the use in the current definition of a fairly direct form of the copy-rule, whereas ref. 6 had exhibited the relation to the VDL definitions of PL/1 (ref. 5) by using the environment mechanism from that model. The existence of such options in the construction of a definition is a result of the use of the constructive definition method and the choices made in the current definition may not be universally liked.

The language defined in Ref. 6 is full Algol (i.e. ref. 1). The current definition is of the ECMA subset plus recursion (see ref. 2). This has two advantages. On the one hand it avoids some of the less clearly defined areas of Algol (e.g. own) and, on the other hand, it defines a language which is more oriented towards static "compilation".

The next Chapter contains a discussion of the main changes made in the current definition with respect to ref. 5. The remainder of the report is a self-contained definition. Chapter 3 describes the notation used in the formal definition. Chapter 4 describes some of the ideas built into the formal definition. Chapter 5 presents, on facing pages, prose and formal descriptions of the language following the structure of ref. 1. The prose description is an amalgam of refs. 1 and 2 with "comments" prompted by ref. 3. The Appendix contains a cross reference index of functions and predicates as an aid to the reader.

The major divisions of the paper have been called Chapters to avoid confusion with the sections of the Algol Report. References into the Algol Report are of the form A.R. followed by a section number. References consisting of only a section number are to the formal definition (i.e. the left hand pages of Chapter 5).

2. DISCUSSION

This section contains a discussion of the main changes to the definition style which have been made with respect to ref. 6.

The Control Component

The control component of a VDL style definition is described in, for example, ref. 4. It is basically a record of what remains to be done in the interpretation. This is achieved by storing instruction names and their arguments in the control. In the VDL definitions the control is made an actual state component. This has two interesting consequences: being a normal object it can be of a tree form; also explicit changes are possible.

The tree form of the control provides a good model for arbitrary ordering between elementary operations. This is achieved by providing a control function (LAMBDA) which selects the next operation from amongst the terminal nodes of the tree. This models in a fairly natural way the arbitrary order of sub-expression evaluation prescribed for Algol.

Without arbitrary ordering the control would exhibit an obvious stack type behaviour; the generalisation in the tree case is also not difficult to envisage. The ability to explicitly change the control complicates this picture.

The jump concept, present in most languages as a local goto (see below for discussion of non-local goto's), has the effect of cutting across the obvious recursive structure of evaluation instructions. This can be modelled by an explicit change to the control whose effect is to delete those entries which corresponded to instructions which looked as though they were to be executed. This explicit change obviously invalidates the simple generalised picture of the stack behaviour of the machine.

"Freezing" of the arbitrary order of evaluation (e.g. entry to a function referenced in expression evaluation) can be modelled by storing away an old control component and installing a new one whose last action will be to re-instate the stored control. Such a model, whilst adequate, causes further distortion of the intuitive stack behaviour since non-local goto commands may now discard stored control components. Another problem caused by storing the old control is that the control can no longer be used to return a value from a function reference (cf. ref. 6).

Perhaps it is unfair to describe the next item as a "consequence", but the possibility of explicitly changing the control can be over-utilized as a short cut. Such a short cut is used in ref. 6 to handle for statements: the result is one of the less clear parts of that definition.

Finally, the necessity to use two levels of function (i.e. the LAMBDA control function and the instructions) was unfortunate, and the description of this (see ref. 5) difficult to understand. It should be admitted that the mechanism now offered is also far from simple. The present authors put more emphasis on a "purely functional" definition and leave the decision on comprehensibility to the reader.

The problems with the control have long been understood (cf. ref. 10). Ideas for modelling the required language features without resort to a control have also been suggested: Ref. 7 shows how the abnormal termination of blocks can be modelled without explicitly changing the control. The basic idea is to anticipate the possibility of an "abnormal" return and to return an indicator which can be tested in the invoking routine. These tests then control the abnormal returns in an orderly fashion rather than goto's "taking the machine by surprise". The problem of the disappointingly large numbers of places where the abnormal value can be returned in the Algol definition is mitigated by the uniformity with which the condition is handled.

The related question of goto commands into structured statements, such as compound and conditional statements, is handled by "cue" functions which set up the appropriate continuation without interpreting those parts of the program which are jumped over.

The problem of representing the required degree of arbitrary ordering in expression evaluation is handled by having a function which accepts the whole expression tree and performs an arbitrary selection of the next operation. Of course, this mirrors the exact role played by the LAMBDA function. The difference, which the present authors consider relevant, is that the tree is part of an abstract program rather than a control containing function names. It is also important to observe that it is only the order of access of variables and function calls which is not defined, the order of evaluation of expressions is defined as left to right. This situation is modelled closely below by having separate functions which access values and which apply operators to values.

Use of the copy rule

Use is made of a form of the copy rule. This results in an extremely natural model of by name parameter passing.

Elimination of the state

VDL models have a state which is global and can be changed by side effect. A fairly mechanical rewriting into functions which have this state as part of their domain and range is possible. It is then clear which parts of the state are not required by or cannot be modified by each function: the domains/ranges can be reduced to only the required items.

A related point is the existence of values of local variables after their blocks cease to exist. Explicit delete operations appear in the current definition which remove the values left in limbo in ref. 6.

Error Detection

There is a class of defects which force a program to be considered invalid for which it is simple to devise static tests (e.g. undeclared variables). If a definition checks for these dynamically the question is left open as to whether a program is in error if such a defect occurs in an "unexecuted" portion. If formal definitions are to provide a reference point for implementations a suitable rule might be "any implementation should produce (one of) the result(s) of the definition unless the latter results in error - in which case the implementation is not further constrained". Clearly such a rule is untenable if compiler checkable (i.e. static) defects such as undeclared variables do not result in error. It is equally clear that one could never define all special cases of errors which are statically checkable. The break-point adopted below is basically to check those things which rely only on symbol matching and omit those checks which, in general, rely on values of symbols. Thus items not statically checked in the current definition are:

Upper bound vs lower bound of arrays
Validity of procedure body after by name substitution
Incompatibility of formal/actual parameters
Applicability of array bounds
Goto into for statement

However, declaration of variables and applicability of operators to their operands is checked by stating the properties of abstract programs which the translator will pass.

Array Values

There is no requirement in Algol to consider array values as structured (cf. ref. 6) . They are treated in the current definition as pairings of integer lists with simple values, where the integer lists correspond to the subscript list.

Abstract Syntax

The abstract program has been made to correspond more closely to the concrete program by leaving labels on the statements and by not using identifiers as selectors. Objects for which identifiers were used as selectors are now shown as sets. This, however, does lead to certain difficulties caused by the lack of text selectors.

Changes to the defined language

Apart from the change to the ECMA subset plus recursion, a few points of detail have been changed from the language defined in ref. 6.

Ref. 6 copies the <array list> onto each array identifier declared. Since this may result in a different number of invocations of any functions referenced from that expected, this decision has not been followed.

Since it is not entirely clear if it is possible to have formal parameters of type procedure passed by value it has been banned in the current definition (see A.R. 4.7.5.4).

There would appear to be no justification for specifying a left to right order of evaluation for subscript expressions occurring other than on the left of an assignment: the restriction of ref. 6 is relaxed in the current definition.

3. NOTATION

Introduction

The definition is a set of functions and predicates which together define the interpretation of any valid ALGOL 60 program. This Chapter describes the notation used to define functions and predicates.

The notation is that of refs. 4 and 11, with slight changes in typography.

In this section, an underlined word indicates that the meaning of that particular term is given at this point.

Objects and Selectors

The basic elements operated on throughout the definition are called objects. Objects may be elementary, that is having no structure, or composite. Composite objects have structure in that they are composed of parts which may be extracted or replaced by the appropriate functions.

The elementary objects used in this definition include members of the following classes:

ids: these are objects used in place of the identifiers of the ALGOL 60 source text,
values: these are the usual arithmetic and boolean values,
the empty list <>: see below under Lists,
sets: various kinds of sets are used; any set is an elementary object, including the empty set { }.

Several other specific elementary objects are used; they are denoted by names spelt in capitals. Thus REAL, INTG, BOOL are the elementary objects corresponding to the ALGOL 60 keywords real, integer, Boolean.

Construction of Composite Objects

Composite objects are constructed out of elementary objects and function names using the function μ_0 as follows. Let OBA, OBB, OBC be three elementary objects to be made parts of a composite object. To extract these three parts of the composite object we need three function names (functions intended for this purpose are given names beginning "s-", and are called

selectors). Let s-1, s-2, s-3 be the three selectors chosen to operate on the object being constructed. These are paired with the elementary object they are to select (notation <s-1:OBA>) and the pairs given to the function μ_0 as arguments, thus:

$$\mu_0(\langle s\text{-}1:OBA\rangle,\langle s\text{-}2:OBB\rangle,\langle s\text{-}3:OBC\rangle) \ .$$

The resulting object, say ob1, has three parts:

$$OBA = s\text{-}1(ob1)$$
$$OBB = s\text{-}2(ob1)$$
$$OBC = s\text{-}3(ob1) \ .$$

Note that this definition of ob1 has also defined new values of each of the functions s-1, s-2 and s-3, namely those given above. (Note also the use of colons as separators in the pairs within the μ_0 arguments. Elsewhere a comma is used for this purpose -- see below under Lists.)

Composite objects may also be used as the second member of an argument to μ_0. Thus if ob2 and ob3 are two further objects, either elementary or composite, then we might define ob4 as:

$$ob4 = \mu_0(\langle s\text{-}a:ob1\rangle,\langle s\text{-}b:ob2\rangle,\langle s\text{-}c:ob3\rangle) \ .$$

Then:

$$s\text{-}a(ob4) = ob1$$
$$s\text{-}b(ob4) = ob2$$
$$s\text{-}c(ob4) = ob3 \ .$$

The arguments to μ_0 may be specified as a set, provided the order in which they are taken does not affect the final object. Thus in the definition of tail (section 1, Lists) we use:

$$\mu_0(\{\langle elem(i):elem(i+1)(list)\rangle \mid 1 \le i \le length(list)\})$$

Here the arguments are the pairs <elem(i):elem(i+1)(list)> where i is in the appropriate range. (Note that the function μ_0 applied to an empty set in this way gives Ω, see below.)

Selector functions, and in fact any functions, may be composed by the functional composition operator, •. The composition of two functions f1 and f2 is defined as follows:

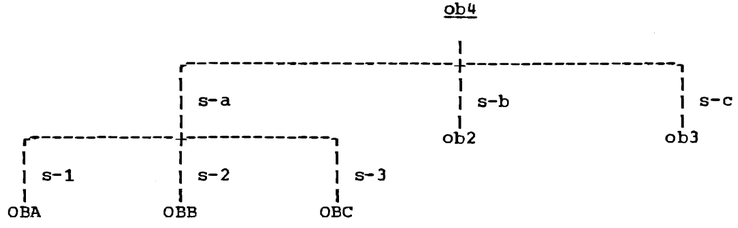
$$f2 \bullet f1(x) = f2(f1(x)) \ .$$

Note that in this use, f1 must be defined for argument x, and if it then returns y, f2 must be defined for this argument. Using this notation, we have from the definitions of ob4 and ob1:

$$s\text{-}1 \bullet s\text{-}a(ob4) = s\text{-}1(s\text{-}a(ob4)) = s\text{-}1(ob1) = OBA \ .$$

Composite selectors are functions formed by composition of selectors in this way. Selectors which are not composite are called elementary selectors.

Composite objects may be thought of as trees, having each branch from one node to another named by a selector, and having elementary objects at each of the terminal nodes. Thus ob4 may be shown diagramatically as a tree as follows.



If ob2 and ob3 are composite, then there will be further structure beneath these nodes.

Values of selector functions for specific arguments thus become defined by the definition of composite objects using them. If a selector is used with an argument for which no value has been defined in this way, then the value is taken to be a special object called Ω. Any selector applied to any elementary object gives Ω (with the sole exception of the identity selector, I, see below). With the example objects defined above:

$$s\text{-}1(ob4) = \Omega$$
$$s\text{-}a(ob1) = \Omega$$
$$s\text{-}a \bullet s\text{-}a(ob4) = \Omega \ .$$

Note that any selector applied to Ω gives Ω, by definition.

The selectors used in the arguments to μ_0 must all be different, otherwise a unique value for the selectors would not be defined if they were applied to the constructed object. For example

ob = μ_0(<s-x:ob1>,<s-x:ob2>,<s-z:ob3>)

does not give a defined value for s-x(ob) -- it might be ob1 or ob2. Hence this formula is invalid. However the same selector may be re-used at a different level of the tree, e.g.:

ob5 = μ_0(<s-1:ob1>,<s-2:ob2>,<s-3:ob3>)

is quite legitimate, giving:

s-1(ob5) = ob1
s-1•s-1(ob5) = OBA .

The identity function, I, is included in the selector functions, since it may be successfully applied to any object. It is defined by:

I(x) = x

-- where x is any argument whatever. When applied to an elementary object it returns that object, not Ω. (This is the only selector that does not give Ω when applied to an elementary object.)

Modification of Objects

Composite objects may be modified by replacing the sub-tree below one node (itself an object) by another object; additions may be made by replacing the empty sub-tree below one node by an object. The function used is μ, with arguments:

i) the object to be modified,
ii) a pair -- as for μ_0 -- consisting of the selector identifying the part of the object to be replaced and the object to be inserted at this point.

For example, with the objects defined above, if

ob6 = μ(ob4;<s-b:ob7>)

ob6 has the structure of ob4 with ob2 (= s-b(ob4)) replaced by ob7. Thus

s-b(ob6) = ob7 .

(Note the semi-colon following the object to be modified.)

Predicates

Meanings are given to the operations & (and), and ∨ (or) of predicate calculus when some of their operands are undefined. In the notation used here (explained below under Definition of Functions) they may be defined as:

x & y =

    cases: x
    FALSE: FALSE
    TRUE: y

x ∨ y =

    cases: x
    TRUE: TRUE
    FALSE: y

The effect of the conditional definitions is that logical expressions become undefined if, working from left to right, an undefined operand is encountered before the value of the expression has been determined. Once a value has been determined, the fact that one or more of the remaining operands is undefined does not make the expression undefined.

The remaining operators are defined as usual in terms of & and ∨ where necessary, using these meanings of & and ∨. The operators are:

¬	not	(undefined if its operand is undefined)
&	and	
∨	or	
⊃	implies	(x ⊃ y = ¬x ∨ y)
≡	is equivalent to	(x ≡ y = (x & y) ∨ (¬x & ¬y))
∃	existential quantifier	
∀	universal quantifier	
i	(iota) see below .	

The i operator is used in the same way as a quantifier, e.g.:

(i x)(is-pred(x)) .

This expression denotes the unique x such that is-pred(x) is true, if one and only one such x exists; otherwise its value is undefined.

In quantified expressions, the bound variable (that following the quantifier symbol in the opening parentheses) is frequently a name that has a corresponding predicate, e.g. (∀sel)(...) with the corresponding predicate is-sel. In such cases the bound variable is assumed to take only values satisfying the predicate. Thus in the example given above, sel is assumed to take only values satisfying is-sel; this is equivalent to adding to the quantified expression inside the second pair of parentheses a term is-sel(sel) & ... If the quantified variable is i or j, then the corresponding predicate is understood to be is-intg-val.

The quantifiers ∀ and ∃ are only used on expressions that are defined for all values of the quantified variable in the range of that variable. This range may be indicated by the name of the variable, as mentioned above; if not, the range is unrestricted.

Predicates are functions whose values are truth-values (either TRUE or FALSE). They may thus be relational expressions, such as x≤1, or expressions built of other predicates using the above operators. Generally they are given names beginning with "is-".

Predicate expressions are sometimes used; they have one of the following forms, defined as below.

(i) (is-pred1 ∨ is-pred2)(x) = is-pred1(x) ∨ is-pred2(x) ,
(ii) (is-pred1 & is-pred2)(x) = is-pred1(x) & is-pred2(x) .

Each elementary object whose name is spelt in capitals has an associated predicate whose name is the name of the object prefixed with is-. Thus

is-REAL(x) = (x = REAL)
is-INTG(x) = (x = INTG) , etc..

For composite objects, many predicates are defined in terms of their selectors and predicates applying to their parts. For example:

```
is-real-op(x) =

    is-REAL•s-type(x) & is-real-val•s-value(x) &
      (is-CONST ∨ is-Ω)•s-const(x) &
      ((sel ≠ s-type & sel ≠ s-value & sel ≠ s-const) ⊃
                                          is-Ω•sel(x))
```

defines is-real-op to be true of x if and only if x has two or three parts, one selected by s-type and satisfying is-REAL (i.e. it is the elementary object REAL), one selected by s-value and satisfying the predicate is-real-val, and a third which may not be present (indicated by the is-Ω

alternative of the predicate) but if it is it must satisfy the predicate is-CONST. Note that x may not have any other parts. This kind of definition is so frequently used that an abbreviated notation is used for it. The general form is:

```
is-x = (<s-a:is-a>,
        <s-b:is-b>,
        ...
        <s-n:is-n>) .
```

This defines is-x to be true of an object if and only if it has parts selected by s-a, s-b, ...,s-n satisfying the predicates is-a,is-b,...,is-n respectively. No other parts may be present; the parts mentioned may be missing if Ω satisfies the relevant predicate. Thus the definition of is-real-op given above takes the form:

```
is-real-op = (<s-type:is-REAL>,
              <s-value:is-real-val>,
              <s-const:is-CONST ∨ is-Ω>) .
```

(See Operands in section 1.) In this style we also use:

is-pred1 = (<is-pred2,is-pred3>)

to define is-pred1 as true only of pairs whose elements satisfy is-pred2 and is-pred3 respectively, and:

is-pred4 = ({is-pred5})

to define is-pred4 as true only of sets whose elements satisfy is-pred5.

Lists

Lists (strictly, non-empty lists) are objects constructed with the special selectors elem(1), elem(2),...,elem(n), where n is the number of elements in the list (the length of the list). They satisfy the predicate:

```
is-non-empty-list = (<elem(1):is-el>,<elem(2):is-el>,...
                                          <elem(n):is-el>)
```

for some n, where is-el is false of Ω, but true of any other object.

An ordering is defined on the elements of a list by the integer arguments to elem. The ith element of a list is

elem(i)(list) .

For i greater than the length of the list, elem(i)(list) = Ω; elem(i) may not be used with i ≤ 0.

The abbreviation:

elem(i,list) for elem(i)(list)

is sometimes used to economise on parentheses.

We also use the notation

<el1,el2,...,eln>

to denote the list with elements el1, el2,...,eln in that order, i.e.

$$\langle el1,el2,...,eln\rangle = \mu_0(\langle elem(1):el1\rangle, \langle elem(2):el2\rangle, \ldots \langle elem(n):eln\rangle)\ .$$

Functions head, giving the first element of a list; tail, giving the list with its first element removed -- and the remaining elements renumbered; length, giving the length of the list are defined (see section 1, Lists). The concatenation operator, ", is also defined between two lists. The empty list, <>, is defined to be an elementary object, since elem(i)(<>) = Ω for any positive i; it satisfies the predicate is-<>, which is false for any other object. It also satisfies the predicate is-list, thus

is-list = is-non-empty-list ∨ is-<> .

Any predicate is-pred has an associated predicate is-pred-list, which is true of <> and of any list all of whose elements satisfy is-pred.

Sets

Sets are elementary objects, since although they have elements no structure is defined on them. A set having specific elements a,b,c,...,n may be defined by:

set1 = {a,b,c,...,n}

-- in which the order of the arguments is immaterial. Thus

{a,b,c} = {b,c,a} = {b,a,c} .

The notation { } represents the empty set, having no elements. We may also define sets using the notation:

set2 = {f(x) | is-pred(x)} .

In this case the set is formed by taking all the objects satisfying is-pred, operating on them with the function f, and putting all the results in the set. (Frequently there will be no f, just x, in such a definition implying that the objects satisfying is-pred themselves are to be the members of the set.)

Any predicate is-pred has an associated predicate is-pred-set which is true of any set all of whose elements satisfy is-pred, and is also true of the empty set { }. The empty set satisfies the predicate is-{ }, which is false for any other object. The predicate is-set is true of any set, including the empty set.

The following operators on sets are used with their usual meaning

∪	set union
−	set difference
∈	is a member of

Paths and Path-els

The existence of sets embedded in the abstract structure of a program, and the lack of selectors to select elements of a set is inconvenient in specifying some of the contextual constraints on a correct program. To overcome this difficulty, functions called path-els are assumed to exist. A path-el is either a selector or a function , like a selector, from a set to one of its elements. It is assumed that, in a given set in the abstract program, each element of the set may be obtained from the set itself by such a path-el function, distinct path-els giving distinct elements of the set. A path is a composition of path-els, so that any part of a given abstract program may be obtained by applying some path to it. Distinct paths give distinct parts of the program.

Definition of Functions

Definitions of functions are given in the normal way, and also by cases using the following basic form:

```
funct(args) =

    cases: f(args)
    is-pred1: def-1
    is-pred2: def-2
    ...
    is-predn: def-n
```

-- where args may be one or more arguments. This form is to be interpreted in the following way. If is-pred1 is true of f(args), then funct(args) is defined by def-1; if is-pred1 is false of f(args) and is-pred2 is true of f(args) then funct(args) is defined by def-2, etc.. If any of the predicates is-predi, i = 1,...,n, is undefined and none of the preceding predicates is true of f(args), then funct(args) is undefined. If all the predicates are false of f(args), then also funct(args) is undefined. Thus functions defined this way may be partial, i.e. they may not be defined for all values of their arguments. In this definition, the interpretation of a program never becomes undefined in this way. If argument values for which a function is not defined are presented to it, then the program must be in error. In such cases, we write a final case giving "error" as the definition. (Occasionally such a definition may appear as a case other than the last, with the same significance.) This indicates that it is an erroneous program being interpreted if this case of the definition is required.

Definitions of this form may be nested within each other. Thus in the above form, def-1 may again be such a definition.

The args given to the function f may be a subset of the arguments of the main function. If the case depends on the value of an argument itself, and not some value constructed from it, then the function f is omitted, and the predicates are then applied to the single argument. If the predicates are testing for equality with a particular elementary object, then we write the object, OB, rather than the corresponding predicate is-OB. The final predicate may be required to be satisfied in all cases not accepted by the previous predicates; in this case the constant predicate T, true for all arguments (and any number of arguments) may be used. Thus the erroneous case mentioned above is written:

```
T:error .
```

For example consider the definition of arithm-prefix-opr of section 3.3:


```
arithm-prefix-opr(op,opr) =

    cases: s-type(op)
    INTG: cases: opr
          PLUS: op
          MINUS: mk-op(INTG, - s-value(op))
    REAL: mk-op(REAL,real-prefix-value(s-value(op),opr))
```

This is constructed as follows. The definition deals with two basic cases, where s-type(op) satisfies is-INTG or is-REAL. Since these predicates test for equality with the elementary objects INTG or REAL, we write these objects in place of the predicates in the basic form. In the first case, i.e. if s-type(op) = INTG then there are two sub-cases. The first applies if the argument opr is the elementary object PLUS. In this case arithm-prefix-opr(op,opr) is defined to be just the object op. In the second sub-case, if the argument opr is the elementary object MINUS, then arithm-prefix-opr(op,opr) is defined to be the value of the function mk-op applied to arguments INTG and -s-value(op). If s-type(op) is REAL then arithm-prefix-op is defined to be the value of the function mk-op applied to arguments REAL and the value of real-prefix-value(s-value(op),opr). The majority of functions in the definition return more than one object; they may be thought of as returning a list of objects. Where such a composite returned value is referred to, we use the notation (el1,el2,...,eln) rather than the usual notation <el1,el2,...,eln> for such a list.

In many function definitions, the cases depend on the values of more than one object returned by another function, or some combination of predicates applied to more than one of the arguments. In such cases, a single predicate or elementary object determining the case may be replaced by a list of predicates or elementary objects. For example in the definition of cue-int-st-list in section 4.1:

```
cue-int-st-list(targ-sel,t,i,dn,vl) =

    cases: cue-int-st(targ-sel,elem(i,t),dn,vl)
    (vl¹,Ω): int-st-list(t,i+1,dn,vl¹)
    (vl¹,lab¹): cases: lab¹
                local(lab¹,t): ...
```

the function cue-int-st returns a list of two objects. The first case of the definition is taken if the second of these is Ω; in this case we use the parameter name vl¹ to refer to the first of the returned objects (whatever it may be) in the following definition of the function. (Note that this use of the parameter vl¹ holds only within this case of the definition. This means of introducing a new name to represent something

occurring in the definition is a special case of the abbreviation by means of let clauses described immediately below.) Here we have used one elementary object (Ω) instead of a predicate is-Ω; the fact that the other name in the case definition is neither an elementary object nor a predicate indicates that the corresponding returned object plays no part in the case distinction.

In many function definitions, abbreviations are used to shorten the text. They are introduced by the word let; following this one or more names are defined as meaning some longer expression or name. Thus in the definition of convert-one-sub in section 3.1:

```
convert-one-sub(eb,op) =

    let: v¹ = convert(INTG,op)
    cases: s-lbd(eb) ≤ v¹ ≤ s-ubd(eb)
    TRUE: v¹
    FALSE: error
```

the v^1 is used as an abbreviation for the expression on the right of the = in the let statement. Again such an abbreviation applies only within the case in which it is defined. Thus in the definition of iterate-for, section 4.6:

```
iterate-for(cvar,for-elem,t,dn,vl) =

    cases: for-elem
    is-arithm-expr:
        cases: eval-expr(for-elem,dn,vl)
        (op¹,vl¹,Ω): let: v = convert(s-type(cvar),op¹)
                          vl² = assign(cvar,v,vl¹)
                     int-st(t,dn,vl²)
        (op¹,vl¹,lab¹): (vl¹,lab¹)
    is-step-until-elem:
        cases: eval-expr(s-init-expr(for-elem),dn,vl)
        (op¹,vl¹,Ω): let: v = convert(s-type(cvar),op¹)
                          vl² = assign(cvar,v,vl¹)
                     iterate-step-until-elem(cvar,
                                             s-step-expr(for-elem),
                                  s-until-expr(for-elem),t,dn,vl²)
        (op¹,vl¹,lab¹): (vl¹,lab¹)
    is-while-elem: iterate-while(cvar,for-elem,t,dn,vl)
```

the abbreviations op^1 and vl^1 are defined differently in the first two cases of for-elem.

In some function definitions, an operation is required to be performed on an element of a set, but which element of the set is arbitrary. (This is necessary if, for instance, the elements of a set are to be operated on in an arbitrary order, not determined by the definition.) Thus the function defined becomes non-deterministic, in that it does not return a uniquely defined result, but rather one of many possible results. A special notation is used to indicate this. For example in the definition of mk-pairs, section 4:

```
mk-pairs(id-set,t,dn) =

    cases: id-set
    { }: { }
    ¬is-{ }: for some id¹ ∈ id-set
             {mk-pair(id¹,t,dn)} ∪ mk-pairs(id-set - {id¹},t,dn)
```

the definition of the second case, where id-set is not empty, requires that some element of id-set is to be used as id^1 in the case definition, but which one is not defined -- any choice will satisfy the definition. In cases such as this, where the set is already given, we use the phrase for some, followed by a requirement on the object denoted by an abbreviation -- in this case that it be an element of the set decl-set.

Each function definition is followed by a clause giving the type of arguments for which it is defined, and the type of values it returns. These types are specified using predicates, which in this use stand for the set of objects satisfying them. The notation

```
type: is-pred1 X is-pred2 → is-pred3 X is-pred4
```

indicates that the function accepts two arguments satisfying is-pred1 and is-pred2 respectively, and returns two objects satisfying is-pred3 and is-pred4 respectively.

4. MECHANISM

The Sequencing Mechanism

The order in which the functions of the definition are evaluated is determined solely by their nesting. Thus int-program (section 4.1) is defined as an application of int-block. In the evaluation of int-block, the function eval-array-decls is evaluated first, since this determines whether or not the values of the other functions will be required. Following this, the functions intr-ids, mk-pairs, change-block, augment-dn, mod-set, int-block-body, seconds, epilogue are evaluated, necessarily in that order since each has as argument the result (or part of the result) of that preceding; the only exception is the pair augment-dn and mod-set, which may be evaluated in either order, since their results are both arguments to int-block-body. The purely functional nature of the definition ensures that the result is the same whichever order is used, since the results of a function evaluation depend only on its arguments, all of which are written explicitly.

Thus the ordering required by the language on the various defined operations must be reflected in the structure of the function definitions.

Two particular styles of definition are used to specify an operation performed on elements of a list in sequence. The simpler one is to define a function - say f1(el,res) - to operate on a single element, and combine this with the results from the preceding elements of the list. This is then used in a recursive definition of a function, say f2, defined as:

```
f2(list,res) =

    cases: list
    <>    : res
    ¬is-<>: let: res' = f1(head(list),res)
            f2(tail(list),res')   .
```

(For example, see iterate-for-list,section 4.6.)

This mechanism does not permit the ordering to depend on the results of any of the functions. In the case of statement sequencing, where exceptions may occur on executing a goto statement, a different mechanism is needed. Here we use an iteration driven by an index to the next statement in the list, as exemplified in int-st-list (section 4.1), and described below.

Normal sequencing through successive statements of a statement list is modelled by int-st-list, with the list and an index, initially 1, as its first two arguments. The effect of execution of the statements is to change the values in vl; int-st-list returns the changed vl as well as an abn component (explained below). The denotations, dn, affect the execution, so dn and the initial values, vl are given to int-st-list as its third and fourth arguments.

Generally, int-st-list has arguments t -- a statement list, i -- the index of a statement in t, dn -- the denotations appropriate to the execution of t, and vl -- the values of known variables after execution of the first i-1 statements of t. The values to be returned are obtained as the result of applying int-st-list to t, i+1, dn, and vl¹, where vl¹ is vl with any changes caused by the execution of the ith statement of t; these are obtained by applying int-st to elem(i,t) -- the ith statement, dn, and vl. The recursion of int-st-list in this way produces a sequence of vl's, giving the changes to values resulting from the execution of the statements of t in normal sequence. The recursion terminates when the new value of i is greater than length(t), and the final vl is returned by int-st-list.

If a goto is executed at some point in the sequence, the appropriate application of int-st will return a label in the abn component of its returned value. The vl resulting from further execution is no longer that obtained by interpreting the remainder of the list. If the label is on a statement within the list, an appropriate vl is obtained from int-st-list with an index to the labelled statement; this is obtained via cue-int-st-list and cue-int-st, which ensure that the mechanism to continue normal sequencing from the labelled statement is set up (the labelled statement may be within a nest of compound statements). If the label is not on a statement of the list, execution of this list is terminated with the current vl, and the label is returned to the functions interpreting the surrounding statement list.

The Evaluation of Expressions

In the evaluation of expressions, the order in which arithmetic and boolean operations are performed is well defined. It is reflected, together with any modifications introduced by parentheses, in the form of the abstract text of the expression. However the order in which subexpressions, e.g. in subscript or actual parameter lists, should be evaluated, and values of variables accessed is not defined.

The formal definition of this situation depends on the following two facts. Firstly, since arithmetic operations do not have any side effects, the order in which they are done relative to the other accessing and evaluation operations cannot affect the final result. Secondly, the resolution of

conditional expressions to the then or else expression, and the resolution of switch designators to the appropriate expression from their switch list, make available further text of the expression; to allow the maximum freedom of choice of ordering, these resolutions should be done as early as possible.

In the definition therefore the choice is made to delay arithmetic and boolean operations as long as possible. The function apply, which deals with these, is thereby given a text in which all operands are ops -- i.e. they have been reduced to values. The function reduce-cond-switch deals with conditionals and switch designators; it is used recursively, since the text introduced by one application of it may contain other conditionals or switch designators which may be resolved immediately. The function access deals with the remaining operations in arbitrary order. These comprise evaluation of the following components of the expression: function references, all of whose by name actual parameters have been fully accessed: simple variables: subscripted variables, all of whose subscripts have been fully accessed: array names (appearing in actual parameter lists). Fully accessed means here that their values can be obtained by apply; this function is used where appropriate to obtain the final value of these parts of the expression.

Static Type Checking

The translation makes use of a function desc (section 1, Miscellaneous), which returns a descriptor from the declarations or specifier from the parameter specifications within whose scope a particular appearance of an identifier lies. The first argument to desc is a path to that appearance of the id; the second is the text which contains this appearance and the declaration or formal parameter specification whose scope contains it. Thus the path applied to the text gives the id. The result depends on the context of an appearance of the id, thus a path to the id within the text is required as an argument rather than the id itself. (The same id may be declared or specified more than once within the text, and different descriptors or specifiers may then be appropriate to different appearances of the same id within the same text).

Procedure Invocations and Function Values

Procedure statements are interpreted by int-proc-st (section 4.7). This function uses access to evaluate the by value actual parameters, and proc-access (section 3.2) to achieve the call. The access mechanism (see the Accessing of Variables above) also uses proc-access (section 3.2).

The function proc-access finishes the evaluation of parameters with a final apply, and passes them to activate-proc. This makes various tests on the

matching of actual and formal parameters, and forms a set of pairs of formal parameter identifiers and their replacements. For by value parameters, the replacement is an identifier - different from the formal parameter identifier if this is already known as a non-local variable - to which the value of the actual parameter is given in vl, and the description of the formal parameter is given in dn. (This is achieved in the test of the actual parameter).

The set of pairs is passed to non-type-proc or type-proc as appropriate. The function type-proc establishes a new identifier to receive the returned value (with type of the procedure being called) in dn and uses insert-ret to modify the relevant appearances of the procedure identifier within its body. It then applies non-type-proc (section 4.7) to proceed with the call.

The function non-type-proc uses the change-text function to insert the replacements for the formal parameters in the procedure body, and passes the modified body to int-proc-body. This function then executes the modified procedure body in the usual way.

Note that the formal parameters are local to the procedure; thus they, or any identifiers by which they were replaced, will be deleted from vl at the epilogue of the procedure. However, the identifier installed for the returned value will not be deleted, since it was not included in the list of such identifiers passed to epilogue by activate-proc. Thus type-proc can extract the type and value of this identifier, and return it through activate-proc and proc-access to fn-access, which will return it into the evaluation of the expression in which the function call occurred.

The Copy Rule

Entry to a procedure involves replacing the by name formal parameters by the text of the corresponding actual parameters. If this text contains identifiers that are declared within the procedure, those declared within the procedure are changed to a different, distinct, identifier. In the definition on entry to any block, the declared identifiers are changed to ids different from those already known (i.e. having entries in the current dn) and from all those declared within any internal blocks or procedure descriptors. (Note that label constants are given entries in dn solely to prevent re-use of their id.) This ensures that no conflicts can occur when these new identifiers appear in by name actual parameters passed to the declared procedures. It should be noted that such substitution for by name formal parameters may give rise to syntactically incorrect text; if this is the case the program is in error.

5. DEFINITION

1 DESCRIPTION OF THE REFERENCE LANGUAGE

1.1 FORMALISM FOR THE DESCRIPTION

Introduction

The syntax of <program> is as given in the Algol Report. A string satisfying this syntax is translated into an object which satisfies is-program as defined under the 'Abstract Syntax' headings below. (The notation used is described in the Notation section.) For the main part the translation is obvious, but the Translator also performs some additional checking. This checking is described under the "Translation" headings by stating properties (in the form of implications) which will hold of any object created by the Translator. As a result of this some programs which satisfy the concrete syntax are rejected and their semantics are not defined.

Some extra predicates are defined in the 'Auxiliary Predicate' sections, and are used as abbreviations for collections of abstract syntax predicates. Note that all predicates defined in the abstract syntax and auxiliary predicate sections begin with is-.

The interpretation of an abstract program is then defined in the 'Interpretation' sections as the application of the function int-program to the translated program. The actual definition given of int-program is a trivial one reflecting the fact that strict Algol 60 programs can only cause any observable effects by calling code procedures. This interpretation corresponds to the semantic description of the Algol 60 language in the sense that it defines the result of interpretation (termination or looping and values available to code procedures), not the way of computing this result.

Appended to the function definitions are the following: a list of references to any predicates or functions not contained in the relevant section or in section 1; a description of any error cases; any explanatory notes required; the type of the defined function shown as a domain and range separated by "→" and stated in terms of cartesian products of (sets satisfying) predicates.

The remainder of this section contains the definitions of some predicates and functions used throughout the formal definition.

INFORMAL DESCRIPTION

This is taken from the 'Revised Report on the Algorithmic Language ALGOL 60' (Ed. P.Naur)

ECMA CHANGES: This version of the report has been changed to reflect the restrictions made for the ECMA subset (see CACM Vol. 6 no. 10 Oct. 1963 pp 595-597). The restriction to non-recursive procedures and the uniqueness of identifiers being controlled only by the first six characters have not been applied. In addition to the changes listed in the reference, sections 2.3, 5.2.1 and 5.2.5 were changed to delete references to own. All changes are marked.

Some examples in the Algol Report use language features not in the ECMA subset. Such examples are marked NON ECMA LANGUAGE.

COMMENT: Where it was felt that the report was open to more than one interpretation (e.g. see Knuth CACM Vol. 10 no. 10) a comment is included to specify the interpretation taken.

DESCRIPTION OF THE REFERENCE LANGUAGE

1. STRUCTURE OF THE LANGUAGE

The algorithmic language has three different kinds of representations - reference, hardware, and publication - and the development described in the sequel is in terms of the reference representation. This means that all objects defined within the language are represented by a given set of symbols - and it is only in the choice of symbols that the other two representations may differ. Structure and content must be the same for all representations.

Objects

(1) is-object =

This predicate is true only of elementary objects (is-set, is-basic-symbol, is-real-val, is-intg-val, is-id, is-<>, and anything of the form is-CAPS. It is false of Ω.

(2) is-ob =

This predicate is true only of elementary or composite objects.

(3) is-selector =

This predicate is true only of elementary selectors. (It is false for I.)

(4) is-sel =

This predicate is true only of elementary or composite selectors (including I).

(5) is-path-el =

A path-el is either a selector or a function from a set to an element of that set. There is some path-el from any set in the abstract form of the program being interpreted to any element of that set.

(6) is-path =

This predicate is true of objects satisfying is-path-el or compositions thereof.

(7) is-set =

This predicate is true of any set, including the empty set, { }.

The purpose of the algorithmic language is to describe computational processes. The basic concept used for the description of calculating rules is the well-known arithmetic expression containing as constituents numbers, variables, and functions. From such expressions are compounded, by applying rules of arithmetic composition, self-contained units of the language - explicit formulae - called assignment statements.

To show the flow of computational processes, certain non-arithmetic statements and statement clauses are added which may describe, e.g. alternatives, or iterative repetitions of computing statements. Since it is necessary for the function of these statements that one statement refers to another, statements may be provided with labels. A sequence of statements may be enclosed between the statement brackets begin and end to form a compound statement.

Statements are supported by declarations which are not themselves computing instructions, but inform the translator of the existence and certain properties of objects appearing in statements, such as the class of numbers taken on as values by a variable, the dimension of an array of numbers, or even the set of rules defining a function. A sequence of declarations followed by a sequence of statements and enclosed between begin and end constitutes a block. Every declaration appears in a block in this way and is valid only for that block.

A program is a block or compound statement which is not contained within another statement and which makes no use of other statements not contained within it.

In the sequel the syntax and semantics of the language will be given.

Whenever the precision of arithmetic is stated as being in general not specified, or the outcome of a certain process is left undefined or said to be undefined, this is to be interpreted in the sense that a program only fully defines a computational process if the accompanying information specifies the precision assumed, the kind of arithmetic assumed, and the course of action to be taken in all such cases as may occur during the execution of the computation.

1.1 FORMALISM FOR SYNTACTIC DESCRIPTION

The syntax will be described with the aid of metalinguistic formulae. Their interpretation is best explained by an example:

<ab> ::= (| [| <ab>(| <ab> <d>

(8) is-[pred]-set(set) =

is-set(set) & (∀el)(el ∈ set ⊃ is-[pred](el))

note: Any defined predicate name, with the initial is- deleted, may be substituted for [pred] in the above definition. For example

is-ob-set(set) = is-set(set) & (∀el)(el ∈ set ⊃ is-ob(el))

is obtained in this way, using the defined predicate is-ob, 1(2).

type: is-ob → is-bool-val

Pairs

(9) is-pr = (<is-ob,is-ob>)

(10) is-idpr = (<is-id,is-id>)

refs: is-id 2.4

(11) s(e,pr-set) =

cases: e
(∃e-1)(<e,e-1> ∈ pr-set): (i e-1)(<e,e-1> ∈ pr-set)
T: error

error: e is not first element of a pair in pr-set. (Only occurs for uninitialised values.)
type: is-ob X is-pr-set → is-ob

(12) del-set(pr-set,ob-set) =

{<e-1,e-2> | <e-1,e-2> ∈ pr-set & ¬(e-1 ∈ ob-set)}

type: is-pr-set X is-ob-set → is-pr-set

(13) mod-set(pr-set-1,pr-set-2) =

{<e-1,e-2> | (<e-1,e-2> ∈ pr-set-2 ∨
(<e-1,e-2> ∈ pr-set-1 & ¬(<e-1,ob> ∈ pr-set-2)))}

type: is-pr-set X is-pr-set → is-pr-set

Sequences of characters enclosed in the brackets <> represent metalinguistic variables whose values are sequences of symbols. The marks ::= and | (the latter with the meaning of or) are metalinguistic connectives. Any mark in a formula, which is not a variable or a connective, denotes itself (or the class of marks which are similar to it). Juxtaposition of marks and/or variables in a formula signifies juxtaposition of the sequences denoted. Thus the fomula above gives a recursive rule for the formation of values of the variable <ab>. It indicates that <ab> may have the value (or [or that given some legitimate value of <ab>, another may be formed by following it with the character (or by following it with some value of the variable <d>. If the values of <d> are the decimal digits, some values of <ab> are:

[(((1(37(
(12345(
(((
[86

In order to facilitate the study, the symbols used for distinguishing the metalinguistic variables (i.e. the sequences of characters appearing within the brackets <> as ab in the above example) have been chosen to be words describing approximately the nature of the corresponding variable. Where words which have appeared in this manner are used elsewhere in the text they will refer to the corresponding syntactic definition. In addition some formulae have been given in more than one place.

Definition:

<empty> ::=
(i.e. the null string of symbols).

(14) firsts(pr-set) =

{ob-1 | <ob-1,ob-2> ∈ pr-set}

type: is-pr-set → is-ob-set

(15) seconds(pr-set) =

{ob-2 | <ob-1,ob-2> ∈ pr-set}

type: is-pr-set → is-ob-set

Denotations

(16) is-type-den = (<s-type:is-type>)

refs: is-type 5.1

(17) is-eb = (<s-lbd:is-intg-val>,
<s-ubd:is-intg-val>)

refs: is-intg-val 2.5

(18) is-array-den = (<s-type:is-type-array>,
<s-bounds:is-eb-list>)

refs: is-type-array 5.2

(19) is-label-den = (<s-type:is-LABEL>,
<s-value:is-id ∨ is-Ω>)

refs: is-id 2.4
note: Label denotations contain a value only for by value parameters and the identifier is that of the actual parameter.

(20) is-switch-den = (<s-switch-list:is-des-expr-list>,
<s-type:is-SWITCH>)

refs: is-des-expr 3.5

(21) is-proc-den = (<s-type:is-type-proc ∨ is-PROC>,
<s-form-par-list:is-id-list>,
<s-spec-pt:is-spec-set>,
<s-value-pt:is-id-set>,
<s-body:is-block ∨ is-code>)

refs: is-type-proc 5.4, is-id 2.4, is-spec 5.4, is-block 4.1, is-code 5.4

(22) is-den = is-type-den ∨ is-array-den ∨ is-proc-den ∨ is-label-den ∨ is-switch-den

(23) is-dn = ({<is-id,is-den>})

refs: is-id 2.4

Values

(24) is-arithm-val = is-real-val ∨ is-intg-val

refs: is-real-val 2.5, is-intg-val 2.5

(25) is-simple-val = is-bool-val ∨ is-arithm-val

refs: is-bool-val 2.2

(26) is-arithm-array-val = ({<is-intg-val-list,is-arithm-val>})

refs: is-intg-val 2.5

(27) is-bool-array-val = ({<is-intg-val-list,is-bool-val>})

refs: is-intg-val 2.5,is-bool-val 2.2

(28) is-array-val = is-arithm-array-val ∨ is-bool-array-val

(29) is-val = is-simple-val ∨ is-array-val

(30) is-vl = ({<is-id,is-val>})

refs: is-id 2.4

Operands

```
(31)   is-arithm-array-op = (<s-type:is-INTG-ARRAY ∨ is-REAL-ARRAY>,
                             <s-bounds:is-eb-list>,
                             <s-value:is-arithm-array-val>)

(32)   is-bool-array-op = (<s-type:is-BOOL-ARRAY>,
                           <s-bounds:is-eb-list>,
                           <s-value:is-bool-array-val>)

(33)   is-array-op = is-arithm-array-op ∨ is-bool-array-op

(34)   is-real-op = (<s-type:is-REAL>,
                     <s-value:is-real-val>,
                     <s-const:is-CONST ∨ is-Ω>)

          refs: is-real-val 2.5


(35)   is-intg-op = (<s-type:is-INTG>,
                     <s-value:is-intg-val>,
                     <s-const:is-CONST ∨ is-Ω>)

          refs: is-intg-val 2.5


(36)   is-bool-op = (<s-type:is-BOOL>,
                     <s-value:is-bool-val>,
                     <s-const:is-CONST ∨ is-Ω>)

          refs: is-bool-val 2.2


(37)   is-arithm-op = is-real-op ∨ is-intg-op

(38)   is-type-op = is-arithm-op ∨ is-bool-op

(39)   is-label-op = (<s-type:is-LABEL>,
                      <s-value:is-id>,
                      <s-const:is-CONST ∨ is-Ω>)

          refs:is-id 2.4


(40)   is-op = is-type-op ∨ is-array-op ∨ is-label-op
```

(41) mk-op(type,v) =

μ_0(<s-type:type>,<s-value:v>)

type: (is-type ∨ is-LABEL) X (is-simple-val ∨ is-id) → is-op

Lists

(42) is-list = is-<> ∨ (<elem(1):is-el>,<elem(2):is-el>,...,<elem(n):is-el>)

note: is-el is true of any object except Ω.

(43) head(list) = elem(1)(list)

type: is-list → is-ob

(44) tail(list) = μ_0({<elem(i):elem(i+1)(list)>|1≤i<length(list)})

type: is-list → is-list

(45) length(list) =

cases: list
<>: 0
¬is-<>: (i j)(elem(j)(list) ≠ Ω & is-Ω(elem(j+1,list)))

type: is-list → is-intg-val

(46) list-1 " list-2 =

μ(list-1;{<elem(length(list-1)+i):elem(i,list-2)> | 1≤i≤length(list-2)})

type: is-list X is-list → is-list

(47) is-[pred]-list(list) =

is-list(list) & (∀i)(1≤i≤length(list) ⊃ is-[pred]•elem(i)(list))

note: Any defined predicate name, with the initial is- deleted, may be substituted for [pred] in the definition above. For example

is-eb-list(list) =

is-list(list) & (∀i)(1≤i≤length(list) ⊃ is-eb•elem(i)(list))

is obtained in this way, using the defined predicate is-eb, 1(17).

type: is-ob → is-bool-val

Builtin functions

(48) sign(x) =

cases: x
x>0: 1
x=0: 0
x<0: -1

type: is-real-val → is-intg-val

(49) entier(x) =

(i i)(is-intg-val(i) & i≤x<i+1)

refs: is-intg-val 2.5
type: is-real-val → is-intg-val

(50) abs(x) = x * sign(x)

type: is-real-val → is-real-val

Miscellaneous

(51) desc(path,t) =

desc-1(path(t),path,t)

note: is-id(path(t))
type: is-path X is-text → (is-specifier ∨ is-desc ∨ is-label-desc)

(52) desc-1(id,path-el•path,t) =

cases: path-el•path(t)
is-proc-desc: desc-proc(id,path-el•path,t)
is-block: desc-block(id,path-el•path,t)
T: desc-1(id,path,t)

refs: is-proc-desc 5.4, desc-proc 5.4, is-block 4.1, desc-block 4.1
type: is-id X is-path X is-program → (is-specifier ∨ is-desc ∨ is-label-desc)

(53) is-abn = is-id ∨ is-Ω

refs: is-id 2.4

(54) is-intr = is-decl ∨ is-label-decl

refs: is-decl 5

(55) is-lab-selector =

One of the elementary selectors s-st-pt, elem(i), s-then-st, s-else-st, or s-st.

(56) is-lab-sel =

Compositions of lab-selectors (including I).

(57) is-label-decl = (<s-id-set:is-id-set>,
<s-desc:is-label-desc>)

refs: is-id 2.4, is-label-desc 5

(58) is-text(t) = (∃p,path)(is-program(p) & is-path(path) & t=path(p))

refs: is-program 4.1

(59) disj(set-1,set-2) =

¬(∃el)(el ∈ set-1 & el ∈ set-2)

type: is-ob-set X is-ob-set → is-bool-val

(60) main-pt(sel) =

(i sel-1)(is-selector(sel-1) & (∃sel-2)(sel-2•sel-1 =sel))

type: is-sel → is-selector

(61) rest-pt(sel) =

(i sel-1)(sel-1•main-pt(sel) =sel)

type: is-sel → is-sel

(62) is-opt-[pred] = is-[pred] ∨ is-Ω

note: Any defined predicate name, with the initial is- deleted, may be substituted for [pred] in the definition above. For example

is-opt-op = is-op ∨ is-Ω

is obtained in this way, using the defined predicate is-op, 1(40).

Translation

(1) All <labels> other than label parameters are translated into label constants.

Abstract syntax

(2) is-basic-symbol =

The set of symbols which are members of the concrete syntax classes <letter>, <digit>, <logical value>, <delimiters>.

(3) is-type-const = is-arithm-const ∨ is-bool-const

refs: is-arithm-const 2.5, is-bool-const 2.2

(4) is-const = is-type-const ∨ is-label-const

refs: is-label-const 3.5

BASIC CONCEPTS

The reference language is built up from the following basic symbols:

<basic symbol> ::= <letter> | <digit> | <logical value> | <delimiter>

2.1 LETTERS

<letter> ::= a|b|c|d|e|f|g|h|i|j|k|l|m|n|o|p|q|r|s|t|u|v|w|x|y|z

ECMA CHANGE

This alphabet may arbitrarily be restricted, or extended with any other distinctive character (i.e. character not coinciding with any digit, logical value or delimiter).

Letters do not have individual meaning. They are used for forming identifiers and strings (cf. sections 2.4 Identifiers, 2.6 Strings).

2.2 DIGITS/LOGICAL VALUES

Abstract syntax

(1) is-bool-val = is-TRUE ᵛ is-FALSE

(2) is-bool-const = (<s-type:is-BOOL>,
<s-value:is-bool-val>,
<s-const:is-CONST>)

2.3 DELIMITERS

Translation

(1) The Translator drops delimiters and comments.

2.2 DIGITS/LOGICAL VALUES

2.2.1 Digits

<digit> ::= 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9

Digits are used for forming numbers, identifiers, and strings.

2.2.2 Logical Values

<logical value> ::= true | false

The logical values have a fixed obvious meaning.

2.3 DELIMITERS

<delimiter> ::= <operator> | <separator> | <bracket> | <declarator> | <specificator>

<operator> ::= <arithmetic operator> | <relational operator> | <logical operator> |
<sequential operator>

<arithmetic operator> ::= + | - | * | / | ÷ | ↑

<relational operator> ::= < | ≤ | = | ≥ | > | ≠

<logical operator> ::= ≡ | ⊃ | ᵛ | & | ¬

<sequential operator> ::= goto | if | then | else | for | do

<seperator> ::= , | . | $_{10}$ | : | ; | := | ␣ | step | until | while | comment

<bracket> ::= (|) | [|] | ‘ | ’ | begin | end

<declarator> ::= Boolean | integer | real | array | switch | procedure

ECMA CHANGE

<specificator> ::= string | label | value

2.4 IDENTIFIERS

Abstract syntax

(1) is-id =

Infinite class of Algol identifiers.

Delimiters have a fixed meaning which for the most part is obvious, or else will be given at the appropriate place in the sequel.

Typographical features such as blank space or change to a new line have no significance in the reference language. They may, however , be used freely for facilitating reading.

For the purpose of including text among the symbols of a program the following "comment" conventions hold:

The sequence of basic symbols:	is equivalent to
;comment <any sequence not containing;>;	;
begin comment <any sequence not containing;>;	begin
end <any sequence not containing end or ; or else>	end.

By equivalence is here meant that any of the three structures shown in the left-hand column may, in any occurrence outside of strings, be replaced by the symbol shown on the same line in the right-hand column without any effect on the action of the program. It is further understood that the comment structure encountered first in the text when reading from left to right has precedence in being replaced over later structures contained in the sequence.

2.4 IDENTIFIERS

2.4.1 Syntax

<identifier> ::= <letter> | <identifier> <letter> | <identifier> <digit>

2.4.2 Examples

q
Soup
V17a
a34kTMNs
MARILYN

2.5 NUMBERS

Abstract syntax

(1) is-real-val =

Real values are as defined in A.R.

(2) is-intg-val =

Integer values are as defined in A.R.

note: It is assumed that is-intg-val(x) ⊃ is-real-val(x).

(3) is-non-neg-intg-val(x) = is-intg-val(x) & x≥0

(4) is-real-const = (<s-type:is-REAL>,
<s-value:is-real-val>,
<s-const:is-CONST>)

(5) is-intg-const = (<s-type:is-INTG>,
<s-value:is-intg-val>,
<s-const:is-CONST>)

(6) is-non-neg-intg-const = (<s-type:is-INTG>,
<s-value:is-non-neg-intg-val>,
<s-const:is-CONST>)

(7) is-arithm-const = is-real-const ∨ is-intg-const

2.4.3 Semantics

Identifiers have no inherent meaning, but serve for the identification of simple variables, arrays, labels, switches, and procedures. They may be chosen freely (cf., however, section 3.2.4 Standard functions).

The same identifier cannot be used to denote two different quantities except when these quantities have disjoint scopes as defined by the declarations of the program (cf. section 2.7 Quantities, kinds and scopes, and section 5 Declarations).

2.5 NUMBERS

2.5.1 Syntax

<unsigned integer> ::= <digit> | <unsigned integer> <digit>

<integer> ::= <unsigned integer> | + <unsigned integer> | - <unsigned integer>

<decimal fraction> ::= . <unsigned integer>

<exponent part> ::= $_{10}$<integer>

<decimal number> ::= <unsigned integer> | <decimal fraction> |
<unsigned integer> <decimal fraction>

<unsigned number> ::= <decimal number> | <exponent part> | <decimal number> <exponent part>

<number> ::= <unsigned number> | + <unsigned number> | - <unsigned number>

2.5.2 Examples

0	-200.084	$-.083_{10}-02$
177	$+07.43_{10}8$	$-_{10}7$
.5384	$9.34_{10}+10$	$_{10}-4$
+0.7300	$2_{10}-4$	$+_{10}+5$

2.5.3 Semantics

Decimal numbers have their conventional meaning. The exponent part is a scale factor expressed as an integral power of 10.

2.6 STRINGS

Abstract syntax

(1) is-string-elem = is-basic-symbol ∨ is-string

refs: is-basic-symbol 2

(2) is-string = is-string-elem-list

2.5.4 Types

Integers are of type integer. All other numbers are of type real (cf. section 5.1 Type declarations).

2.6 STRINGS

2.6.1 Syntax

<proper string> ::= <any sequence of basic symbols not containing ␣ or ␣> | <empty>

<open string> ::= <proper string> | ␣<open string>␣ | <open string> <open string>

<string> ::= ␣<open string>␣

2.6.2 Examples

␣5k,,-␣[[[␣&=/:T␣t␣␣
␣..This␣is␣a␣␣string␣␣

2.6.3 Semantics

In order to enable the language to handle arbitrary sequences of basic symbols the string quotes ␣ and ␣ are introduced. The symbol ␣ denotes a space. It has no significance outside strings.

Strings are used as actual parameters of procedures (cf. sections 3.2 Function designators and 4.7 Procedure statements).

2.7 QUANTITIES, KINDS AND SCOPES

The following kinds of quantities are distinguished: simple variables, arrays, labels, switches, and procedures.

The scope of a quantity is the set of statements and expressions in which the declaration of the identifier associated with that quantity is valid. For labels see section 4.1.3.

2.8 VALUES AND TYPES

A value is an ordered set of numbers (special case: a single number), an ordered set of logical values (special case: a single logical value), or a label.

Certain of the syntactic units are said to possess values. These values will in general change during the execution of the program. The values of expressions and their constituents are defined in section 3. The value of an array identifier is the ordered set of values of the corresponding array of subscripted variables (cf. section 3.1.4.1).

The various types (integer, real, Boolean) basically denote properties of values. The types associated with syntactic units refer to the values of these units.

Notes

In order to avoid the necessity of defining a separate class of objects to cover the changed expressions generated by interpretation, the definition of is-expr permits operands to occur as primitive elements.

Translation

(1) Only constants (a subset of operands) can occur:

is-expr(e) & is-sel(sel) & is-op•sel(e) ⊃ is-const•sel(e)

refs: is-const 2

Abstract syntax

(2) is-expr = is-arithm-expr ∨ is-bool-expr ∨ is-des-expr

refs: is-arithm-expr 3.3, is-bool-expr 3.4, is-des-expr 3.5

Auxiliary predicates

(3) is-op-expr(t) =

is-expr(t) & (¬(∃sel)((is-funct-ref ∨ is-var ∨ is-switch-des)(sel(t))) &
(is-cond-expr•sel(t) ⊃ is-op•s-decision•sel(t) &
(is-TRUE•s-value•s-decision•sel(t) ⊃ is-op-expr•s-then-expr•sel(t)) &
(is-FALSE•s-value•s-decision•sel(t) ⊃ is-op-expr•s-else-expr•sel(t))))

refs: is-funct-ref 3.2, is-var 3.1, is-switch-des 3.5, is-cond-expr 3.3

3. EXPRESSIONS

In the language the primary constituents of the programs describing algorithmic processes are arithmetic, Boolean, and designational, expressions. Constituents of these expressions, except for certain delimiters, are logical values, numbers, variables, function designators, and elementary arithmetic, relational, logical, and sequential, operators. Since the syntactic definition of both variables and function designators contains expressions, the definition of expressions, and their constituents, is necessarily recursive.

COMMENT: The order in which the primaries are referenced within expressions is taken to be arbitrary. Since the evaluation of function designators may cause a change in the value designated by a variable (i.e. side-effects), the value of an expression may not be unique.

<expression> ::= <arithmetic expession> | <Boolean expression> | <designational expression>

Interpretation

```
(4)   eval-expr(t,dn,vl) =

          cases: t
          is-funct-ref:  cases: access(t,dn,vl)
                         (t¹,vl¹,Ω): fn-access(t¹,dn,vl¹)
                         (t¹,vl¹,lab¹): (Ω,vl¹,lab¹)
          ¬is-funct-ref: cases: access(t,dn,vl)
                         (t²,vl²,Ω): (apply(t²),vl²,Ω)
                         (t²,vl²,lab²): (Ω,vl²,lab²)

      refs: is-funct-ref 3.2, fn-access 3.2
      note: The case distinction is made because access will not invoke the final function
            reference(cf. note on access).
      type: is-expr X is-dn X is-vl → is-opt-op X is-vl X is-abn


(5)   access(t,dn,vl) =

          let: t¹ = reduce-cond-switch(t,dn)
          cases: (t¹,ready-set(t¹,dn))
          ((is-funct-ref ∨ is-proc-st),{I}):
               (t¹,vl,Ω)
          (¬(is-funct-ref ∨ is-proc-st),{ }):
               (t¹,vl,Ω)
          T:   for some sel¹ ∈ ready-set(t¹,dn)
               cases: one-access(sel¹(t¹),dn,vl)
               (t²,vl²,Ω): access(μ(t¹;<sel¹:t²>),dn,vl²)
               (t²,vl²,lab²): (Ω,vl²,lab²)

      refs: is-funct-ref 3.2, is-proc-st 4.7
      note: Since this function handles procedure statements the final access is handled as a
            special case in order to avoid the check for no returned value.
      type: is-text X is-dn X is-vl → is-opt-text X is-vl X is-abn
```

```
(6)    reduce-cond-switch(t,dn) =

           cases: t
           is-op ∨ is-object ∨ is-simple-var: t
           is-funct-ref:
                cases: s-act-par-list(t)
                Ω: t
                ¬is-Ω: μ(t;{<elem(i)•s-act-par-list:
                                reduce-cond-switch(elem(i)•s-act-par-list(t),dn)> |
                                1≤i≤length•s-act-par-list(t) & is-value-parm(i,s(s-id(t),dn))})
           is-switch-des:
                let: sub-list¹ = reduce-cond-switch(s-subscr-list(t),dn)
                cases: sub-list¹
                ¬is-op-expr-list: μ(t;<s-subscr-list:sub-list¹>)
                is-op-expr-list:
                     let: v¹ = convert(INTG,apply•elem(1,sub-list¹))
                          list¹ = s-switch-list•s(s-id(t),dn)
                     cases: v¹
                     1≤v¹≤length(list¹):
                          reduce-cond-switch(elem(v¹,list¹),dn)
                     T: error
           is-cond-expr:
                let: dec² = reduce-cond-switch(s-decision(t),dn)
                cases: dec²
                ¬is-op-expr: μ(t;<s-decision:dec²>)
                is-op-expr:
                     let: op² = apply(dec²)
                     cases: s-value(op²)
                     TRUE: μ(t;<s-decision:op²>,
                               <s-then-expr:reduce-cond-switch(s-then-expr(t),dn)>)
                     FALSE: μ(t;<s-decision:op²>,
                                <s-else-expr:reduce-cond-switch(s-else-expr(t),dn)>)
           is-ob: μ₀({<sel:reduce-cond-switch(sel(t),dn)> | is-selector(sel) & ¬is-Ω•sel(t)})

      refs: is-simple-var 3.1, is-funct-ref 3.2, is-switch-des 3.5, convert 4.2,
            is-cond-expr 3.3
      type: is-text X is-dn → is-text
```

```
(7)    ready-set(t,dn) =

           cases: t
           is-object ∨ is-op: {}
           is-simple-var ∨ is-array-name: {I}
           is-subscr-var:
                let: set¹ = ready-set(s-subscr-list(t),dn)
                cases: set¹
                {}: {I}
                ¬is-{}: {sel•s-subscr-list | sel ∈ set¹}
           is-funct-ref:
                cases:s-act-par-list(t)
                Ω: {I}
                ¬is-Ω: let: set² = {sel•elem(i) | sel∈ready-set(elem(i)•s-act-par-list(t),dn) &
                                                  1≤i≤length•s-act-par-list(t) &
                                                  is-value-parm(i,s(s-id(t),dn))}
                       cases: set²
                       {}: {I}
                       ¬is-{}: {sel•s-act-par-list | sel ∈ set²}
           is-cond-expr:
                cases: s-decision(t)
                ¬is-op: {sel•s-decision | sel ∈ ready-set(s-decision(t),dn)}
                is-op: cases: s-value(s-decision(t))
                       TRUE: {sel•s-then-expr | sel ∈ ready-set(s-then-expr(t),dn)}
                       FALSE: {sel•s-else-expr | sel ∈ ready-set(s-else-expr(t),dn)}
           is-ob: {sel•sel-1 | ¬is-Ω•sel-1(t) & is-selector(sel-1) &
                               sel ∈ ready-set(sel-1(t),dn)}

      refs: is-simple-var 3.1, is-array-name 3.2, is-subscr-var 3.1, is-funct-ref 3.2,
            is-cond-expr 3.3
      type: is-text X is-dn → is-sel-set


(8)    one-access(t,dn,vl) =

           cases: t
           is-funct-ref: fn-access(t,dn,vl)
           is-simple-var: (simp-var-access(t,dn,vl),vl,Ω)
           is-subscr-var: (subscr-var-access(t,dn,vl),vl,Ω)
           is-array-name: (array-access(t,dn,vl),vl,Ω)

      refs: is-funct-ref 3.2, fn-access 3.2, is-simple-var 3.1, simp-var-access 3.1,
            is-subscr-var 3.1, subscr-var-access 3.1, is-array-name 3.2, array-access 3.1
      type: is-text X is-dn X is-vl → is-opt-op X is-vl X is-abn
```

```
(9)   apply(e) =

          cases: e
          is-op: e
          is-arithm-expr: apply-arithm-opr(e)
          is-bool-expr: apply-bool-opr(e)
          is-des-expr: apply-des-opr(e)

      refs: is-arithm-expr 3.3, apply-arithm-opr 3.3, is-bool-expr 3.4, apply-bool-opr 3.4,
            is-des-expr 3.5, apply-des-opr 3.5
      type: (is-expr ∨ is-array-op) → is-op


(10)  is-value-parm(i,den) =

          elem(i)•s-form-par-list(den) ∈ s-value-pt(den)

      type: is-intg-val X is-proc-den → is-bool-val
```


Translation

All variable identifiers must be declared in declarations or specifications:

```
(1)   is-path(path) & is-real-simple-var•path(prog) ⇒ is-REAL•desc(s-id•path,prog)

(2)   is-path(path) & is-intg-simple-var•path(prog) ⇒ is-INTG•desc(s-id•path,prog)

(3)   is-path(path) & is-bool-simple-var•path(prog) ⇒ is-BOOL•desc(s-id•path,prog)

(4)   is-path(path) & is-real-subscr-var•path(prog) ⇒ (is-REAL-ARRAY•s-type•desc(s-id•path,prog) ∨
                                                        is-REAL-ARRAY•desc(s-id•path,prog))

(5)   is-path(path) & is-intg-subscr-var•path(prog) ⇒ (is-INTG-ARRAY•s-type•desc(s-id•path,prog) ∨
                                                        is-INTG-ARRAY•desc(s-id•path,prog))

(6)   is-path(path) & is-bool-subscr-var•path(prog) ⇒ (is-BOOL-ARRAY•s-type•desc(s-id•path,prog) ∨
                                                        is-BOOL-ARRAY•desc(s-id•path,prog))
```

Abstract Syntax

(7) is-real-simple-var = (<s-id:is-id>,
<s-type:is-REAL>)

refs: is-id 2.4

(8) is-real-subscr-var = (<s-id:is-id>,
<s-subscr-list:(is-arithm-expr-list & ¬is-<>)>,
<s-type:is-REAL>)

refs: is-id 2.4, is-arithm-expr 3.3

(9) is-intg-simple-var = (<s-id:is-id>,
<s-type:is-INTG>)

refs:is-id 2.4

(10) is-intg-subscr-var = (<s-id:is-id>,
<s-subscr-list:(is-arithm-expr-list & ¬is-<>)>,
<s-type:is-INTG>)

refs: is-id 2.4, is-arithm-expr 3.3

(11) is-bool-simple-var = (<s-id:is-id>,
<s-type:is-BOOL>)

refs: is-id 2.4

(12) is-bool-subscr-var = (<s-id:is-id>,
<s-subscr-list:(is-arithm-expr-list & ¬is-<>)>,
<s-type:is-BOOL>)

refs: is-id 2.4, is-arithm-expr 3.3

(13) is-real-var = is-real-simple-var ∨ is-real-subscr-var

(14) is-intg-var = is-intg-simple-var ∨ is-intg-subscr-var

(15) is-arithm-var = is-real-var ∨ is-intg-var

3.1.1 Syntax

<variable identifier> ::= <identifier>

<simple variable> ::= <variable identifier>

<subscript expression> ::= <arithmetic expression>

<subscript list> ::= <subscript expression> | <subscript list> , <subscript expression>

<array identifier> ::= <identifier>

<subscripted variable> ::= <array identifier> [<subscript list>]

<variable> ::= <simple variable> | <subscripted variable>

3.1.2 Examples

```
epsilon
detA
a17
Q[7,2]
x[sin(n*pi/2),Q[3,n,4]]
```

(16) is-bool-var = is-bool-simple-var ∨ is-bool-subscr-var

Auxiliary Predicates

(17) is-simple-var = is-real-simple-var ∨ is-intg-simple-var ∨ is-bool-simple-var ∨ is-label-var

refs: is-label-var 3.5

(18) is-subscr-var = is-real-subscr-var ∨ is-intg-subscr-var ∨ is-bool-subscr-var

(19) is-var = is-simple-var ∨ is-subscr-var

Interpretation

(20) simp-var-access(t,dn,vl) =

cases: s-type(t)
LABEL: mk-op(LABEL,s-value•s(s-id(t),dn))
is-type: mk-op(s-type(t),s(s-id(t),vl))

refs: is-type 5.1
type: is-simple-var X is-dn X is-vl → (is-type-op ∨ is-label-op)

(21) subscr-var-access(t,dn,vl) =

let: $subl^1$ = μ_0({<elem(i):apply•elem(i)•s-subscr-list(t)> | 1≤i≤length•s-subscr-list(t)})
$subl^2$ = convert-subs(s-bounds•s(s-id(t),dn),$subl^1$)
mk-op(s-type(t),s($subl^2$,s(s-id(t),vl)))

refs: apply 3
type: is-subscr-var X is-dn X is-vl → is-type-op

3.1.3 Semantics

A variable is a designation given to a single value. This value may be used in expressions for forming other values and may be changed at will by means of assignment statements (section 4.2). The type of the value of a particular variable is defined in the declaration for the variable itself (cf. section 5.1 Type declarations) or for the corresponding array identifier (cf. section 5.2 Array declarations).

3.1.4 Subscripts

3.1.4.1 Subscripted variables designate values which are components of multidimensional arrays (cf. section 5.2 Array declarations). Each arithmetic expression of the subscript list occupies one subscript position of the subscripted variable, and is called a subscript. The complete list of subscripts is enclosed in the subscript brackets []. The array component referred to by a subscripted variable is specified by the actual numerical value of its subscripts (cf. section 3.3 Arithmetic expressions).

3.1.4.2 Each subscript position acts like a variable of type integer and the evaluation of the subscript is understood to be equivalent to an assignment to this fictitious variable (cf. section 4.2.4).

COMMENT: The order in which references within subscripts are made is not constrained. (see 3, also cf. 4.2.3.1)

The value of the subscripted variable is defined only if the value of the subscript expression is within the subscript bounds of the array (cf. section 5.2 Array declarations).

```
(22)  convert-subs(eb-list,sub-list) =

          cases: length(eb-list) = length(sub-list)
          TRUE: {<elem(i):convert-one-sub(elem(i,eb-list),elem(i,sub-list))> |
                                                              1≤i≤length(eb-list)}
          FALSE: error

      error: If lengths of bound list and subscript list are unequal.
      type: is-eb-list X is-arithm-op-list → is-intg-val-list


(23)  convert-one-sub(eb,op) =

          let: v¹ = convert(INTG,op)
          cases: s-lbd(eb)≤v¹≤s-ubd(eb)
          TRUE: v¹
          FALSE: error

      refs: convert 4.2
      error: If the subscript value is outside the bounds.
      type: is-eb X is-arithm-op → is-intg-val


(24)  array-access(arr-name,dn,vl) =

          let: id = s-id(arr-name)
               ebl = s-bounds(s(id,dn))
               subl-set¹ = {<j(1),j(2),...,j(n)> | n = length(ebl) &
                    (1≤i≤n ⊃ is-intg-val•j(i) & s-lbd•elem(i)(ebl)≤j(i)≤s-ubd•elem(i)(ebl))}
          cases: (∀subl)(subl ∈ subl-set¹ ⊃ (∃val)(<subl,val> ∈ s(s-id(t),vl)))
          TRUE: μ0(<s-bounds:s-bounds•s(s-id(t),dn)>,
                   <s-value:s(s-id(t),vl)>,
                   <s-type:s-type(t)>)
          FALSE: error

      refs: is-intg-val 2.5
      error: If any element of the array is not initialised.
      type: is-array-name X is-dn X is-vl → is-array-op
```

Translation

Function designators are characterised as follows:

(1) is-path(path) & (is-real-funct-ref(path(prog)) ⊃
is-REAL-PROC(s-type•desc(s-id•path,prog)) ∨ is-REAL-PROC(desc(s-id•path,prog))

(2) is-path(path) & (is-intg-funct-ref(path(prog)) ⊃
is-INTG-PROC(s-type•desc(s-id•path,prog)) ∨ is-INTG-PROC(desc(s-id•path,prog))

(3) is-path(path) & (is-bool-funct-ref(path(prog)) ⊃
is-BOOL-PROC(s-type•desc(s-id•path,prog)) ∨ is-BOOL-PROC(desc(s-id•path,prog))

(4) is-path(path) & is-non-type-proc-name•path(prog) ⊃
is-PROC•s-type•desc(s-id•path,prog) ∨ is-PROC•desc(s-id•path,prog)

(5) Unsubscripted array variables are only permitted in actual parameter lists:

is-path(path) & is-array-name•path(prog) ⊃
(∃path-1,i)(path = elem(i)•s-act-par-list•path-1) &
is-type-array•desc(s-id•path,prog) ∨ is-type-array•s-type•desc(s-id•path,prog)

refs: is-type-array 5.2

Abstract syntax

(6) is-non-type-proc-name = (<s-id:is-id>,
<s-type:is-PROC>)

refs: is-id 2.4

(7) is-array-name = (<s-id:is-id>,
<s-type:is-type-array>)

refs: is-id 2.4, is-type-array 5.2

(8) is-act-par = is-expr ∨ is-array-name ∨ is-non-type-proc-name ∨ is-string

refs: is-expr 3, is-string 2.6
note: Procedure identifiers referring to type procedures satisfy is-expr.

3.2.1 Syntax

<procedure identifier> ::= <identifier>

<actual parameter> ::= <string> | <expression> | <array identifier> | <switch identifier> |
<procedure identifier>

<letter string> ::= <letter> | <letter string> <letter>

<parameter delimeter> ::= , |) <letter string> : (

<actual parameter list> ::= <actual parameter> |
<actual parameter list> <parameter delimeter> <actual parameter>

<actual parameter part> ::= <empty> | (<actual parameter list>)

<function designator> ::= <procedure identifier> <actual parameter part>

(9) is-real-funct-ref = (<s-id:is-id>,
<s-act-par-list:is-act-par-list ∨ is-Ω>,
<s-type:is-REAL-PROC>)

refs: is-id 2.4

(10) is-intg-funct-ref = (<s-id:is-id>,
<s-act-par-list:is-act-par-list ∨ is-Ω>,
<s-type:is-INTG-PROC>)

refs: is-id 2.4

(11) is-bool-funct-ref = (<s-id:is-id>,
<s-act-par-list:is-act-par-list ∨ is-Ω>,
<s-type:is-BOOL-PROC>)

refs: is-id 2.4

Auxiliary predicates

(12) is-funct-ref = is-real-funct-ref ∨ is-intg-funct-ref ∨ is-bool-funct-ref

(13) is-fn-ret = (<s-type:is-type>,
<s-value:is-simple-val ∨ is-Ω>)

refs: is-type 5.1

Interpretation

(14) fn-access(t,dn,vl) =

cases: proc-access(t,dn,vl)
(op¹,vl¹,Ω): cases: op¹
¬is-Ω: (op¹,vl¹,Ω)
Ω: error
(op¹,vl¹,lab¹): (Ω,vl¹,lab¹)

error: If a procedure (invoked by a function reference) terminates normally but does not return a value.
type: is-funct-ref X is-dn X is-vl → is-opt-op X is-vl X is-abn

3.2.2 Examples

sin(a-b)
J(v+s,n)
R
S(s-5) Temperature: (T) Pressure: (P)
Compile(ι:=J) Stack: (Q)

3.2.3 Semantics

Function designators define single numerical or logical values, which result through the application of given sets of rules defined by a procedure declaration (cf. section 5.4 Procedure declarations) to fixed sets of actual parameters. The rules governing specification of actual parameters are given in section 4.7 Procedure statements. Not every procedure declaration defines the value of a function designator.

3.2.4 Standard functions

Certain identifiers should be reserved for the standard functions of analysis, which will be expressed as procedures. It is recommended that this reserved list should contain:

```
(15)   proc-access(t,dn,vl) =

       let: act-par-list¹ = cases: s-act-par-list(t)
                            Ω: <>
                            ¬is-Ω: μ(s-act-par-list(t);
                                     {<elem(i):apply•elem(i)•s-act-par-list(t)> |
                                        1≤i≤length•s-act-par-list(t) &
                                        is-value-parm(i,s(s-id(t),dn))})
       activate-proc(s(s-id(t),dn),s-id(t),s-type(t),act-par-list¹,dn,vl)

   refs: apply 3, is-value-parm 3
   type: (is-funct-ref ∨ is-proc-st) X is-dn X is-vl → is-opt-op X is-vl X is-abn
```


Unrestricted

abs(E)	for the modulus (absolute value) of the value of the expression E
sign(E)	for the sign of the value of E (+1 for E>0, 0 for E=0, -1 for E<0)
sqrt(E)	for the square root of the value of E
sin(E)	for the sine of the value of E
cos(E)	for the cosine of the value of E
arctan(E)	for the principal value of the arctangent of the value of E
ln(E)	for the natural logarithm of the value of E
exp(E)	for the exponential function of the value of E (e ↑ E).

These functions are all understood to operate indifferently on arguments both of type real and integer. They will all yield values of type real, except for sign(E) which will have values of type integer. In a particular representation these functions may be available without explicit declarations (cf. section 5 Declarations).

3.2.5 Transfer functions

It is understood that transfer functions between any pair of quantities and expressions may be defined. Among the standard functions it is recommended that there be one, namely

entier(E) ,

which "transfers" an expression of real type to one of integer type, and assigns to it the value which is the largest integer not greater than the value of E.

```
(16)  activate-proc(proc,id,type,act-par-list,dn,vl) =

        let: form-par¹(i) = elem(i,s-form-par-list(proc))
             length¹ = length(s-form-par-list(proc))
        cases: length(act-par-list) = length¹
        FALSE: error
        TRUE:
             let: conv-act-par-list = eval-act-par-list(act-par-list,s-form-par-list(proc),
                                                        s-spec-pt(proc),s-value-pt(proc))
                  act-par¹(i) = elem(i,conv-act-par-list)
                  name-repl = {<form-par¹(i),act-par¹(i)> |
                                   1≤i≤length¹ & ¬is-value-parm(i,proc)}
                  val-repl = mk-pairs({form-par¹(i) |
                                1≤i≤length¹ & is-value-parm(i,proc)},proc,dn)
                  dn¹ = mod-set(dn,{<id²,val-den(act-par¹(i))> |
                                1≤i≤length¹ & <id¹,id²> ∈ val-repl & id¹ = form-par¹(i)})
                  vl¹ = mod-set(vl,{<id²,s-value(act-par¹(i))> |
                                1≤i≤length¹ & <id¹,id²> ∈ val-repl &
                                ¬is-label-op(act-par¹(i)) & id¹ = form-par¹(i)})
             cases: type
             is-PROC: cases: non-type-proc(s-body(proc),val-repl ∪ name-repl,dn¹,vl¹)
                      (vl²,Ω): (Ω,epilogue(seconds(val-repl),vl²),Ω)
                      (vl²,lab²): (Ω,epilogue(seconds(val-repl),vl²),lab²)
             is-type-proc: cases: type-proc(proc,id,type,val-repl ∪ name-repl,dn¹,vl¹)
                           (op²,vl²,Ω): (op²,epilogue(seconds(val-repl),vl²),Ω)
                           (op²,vl²,lab²): (Ω,epilogue(seconds(val-repl),vl²),lab²)

   refs: is-value-parm 3, mk-pairs 4.1, non-type-proc 4.7, epilogue 4.1, is-type-proc 5.4
   error: If lengths of actual and formal parameter lists are unequal.
   note: This function called in both procedure and functional cases of procedure
         invocation.
   type: is-proc-den X is-id X is-type-proc X is-act-par-list X is-dn X is-vl →
                                                          is-opt-op X is-vl X is-abn


(17)  eval-act-par-list(act-par-list,form-par-list,spec-pt,val-pt) =

        let: spec¹(i) = (i spec)(spec ∈ spec-pt & s-id(spec) = elem(i,form-par-list))
        μ₀({<elem(i):eval-act-par(elem(i,act-par-list),s-specifier(spec¹(i)),
                          (elem(i,form-par-list),val-pt))> | i≤i≤length(form-par-list)})

   type: is-act-par-list X is-id-list X is-spec-set X is-id-set → is-act-par-list
```

```
(18)   eval-act-par(act-par,spec,flag) =

            cases: flag
            TRUE: cases: spec
                  is-arithm: cases: act-par
                             is-arithm-op: let: val = convert(spec,act-par)
                                           mk-op(spec,val)
                             ¬is-arithm-op: error
                  is-BOOL:cases: act-par
                          is-bool-op: act-par
                          ¬is-bool-op: error
                  is-arithm-array:
                      cases: act-par
                      is-arithm-array-op: let: val = convert-array(spec,act-par)
                                          μ(act-par;<s-type:spec>,
                                                    <s-value:val>)
                      ¬is-arithm-array-op: error
                  is-BOOL-ARRAY:
                      cases: act-par
                      is-bool-array-op: act-par
                      ¬is-bool-array-op: error
                  is-LABEL: cases: act-par
                            is-label-op: act-par
                            ¬is-label-op: error
            FALSE: cases: match(spec,act-par)
                   TRUE: act-par
                   FALSE: error

      refs: is-arithm 5.1, convert 4.2, is-arithm-array 5.2
      error: If act-par and spec do not conform.
      note: flag is TRUE if and only if act-par is passed by value.
            See also 5.4(8) for constraints on by value specifications.
      type: is-act-par X is-specifier X is-bool-val → is-act-par


(19)   convert-array(spec,act-par) =

          {<int-list,val> | <int-list,v> ∈ s-value(act-par) &
                            val = convert-array-el(spec,v)}

      type: is-arithm-array X is-arithm-array-op → is-arithm-array-val
```

```
(20)   convert-array-el(arr-type,val) =

           cases: (arr-type,val)
           (REAL-ARRAY,is-intg-val): val
           (INTG-ARRAY,is-real-val): entier(val+0.5)
           T: val

       type: is-arithm-array X is-arithm-val → is-arithm-val


(21)   match(spec,act-par) =

           is-STRING(spec) & is-string(act-par) ∨
           is-INTG(spec) & is-intg-expr(act-par) ∨
           is-REAL(spec) & is-real-expr(act-par) ∨
           is-BOOL(spec) & is-bool-expr(act-apr) ∨
           is-type-array(spec) & spec = s-type(act-par) ∨
           is-LABEL(spec) & is-des-expr(act-par) ∨
           is-SWITCH(spec) & s-type(act-par) = spec ∨
           is-PROC(spec) & spec = s-type(act-par) ∨
           is-type-proc(spec) & spec = s-type(act-par)

       refs: is-string 2.6, is-intg-expr 3.3, is-real-expr 3.3, is-bool-expr 3.4,
             is-type-array 5.2, is-des-expr 3.5, is-type-proc 5.4
       type: is-specifier X is-act-par → is-bool-val


(22)   val-den(act-par) =

           cases: act-par
           is-type-op: μ0(<s-type:s-type(act-par)>)
           is-array-op: μ0(<s-type:s-type(act-par)>,
                           <s-bounds:s-bounds(act-par)>)
           is-label-op: act-par

       type: is-op → is-den
```

```
(23)  type-proc(den,id,type,pr-set,dn,vl) =

          let: {<id,id¹>} = mk-pairs({id},den,dn)
               type¹ = cases: type
                       REAL-PROC: REAL
                       INTG-PROC: INTG
                       BOOL-PROC: BOOL
               t¹ = insert-ret(id,id¹,type¹,s-body(den))
               dn¹ = mod-set(dn,{<id¹,μ0(<s-type:type¹>)>})
          cases: non-type-proc(t¹,pr-set,dn¹,vl)
          (vl¹,Ω): cases: (∃v)(<id¹,v> ∈ vl¹)
                   TRUE: (μ0(<s-type:type¹>,<s-value:s(id¹,vl¹)>),epilogue({id¹},vl¹),Ω)
                   FALSE: (μ0(<s-type:type¹>,<s-value:Ω>),epilogue({id¹},vl¹)Ω)
          (vl¹,lab¹): (Ω,epilogue({id¹},vl¹),lab¹)

     refs: mk-pairs 4.1, non-type-proc 4.7, epilogue 4.1
     note: func-id is made into a var with ret-id as its s-id component.
           Although no value is being returned from a type procedure, error should not be
           given unless invoked as a function reference (see fn-access).
     type: is-proc-den X is-id X is-type-proc X is-pr-set X is-dn X is-vl →
                                                  is-opt-fn-ret X is-vl X is-abn


(24)  insert-ret(func-id,ret-id,type,t) =

          cases: t
          is-code: t
          is-assign-st: μ(t;{<elem(i)•s-lp:μ0(<s-id:ret-id>,<s-type:type>)> |
                                  1≤i≤length(s-lp(t)) & is-activated-fn(elem(i,s-lp(t))) &
                                  s-id(elem(i,s-lp(t)))= func-id})
          is-set: {insert-ret(func-id,ret-id,type,el) | el ∈ t}
          is-object: t
          is-ob: μ0({<sel:insert-ret(func-id,ret-id,type,sel(t))> | is-selector(sel) &
                                                                    ¬is-Ω(sel(t))})

     refs: is-code 5.4, is-assign-st 4.2, is-activated-fn 4.2
     type: is-id X is-id X is-type X is-text → is-text
```

Translation

(1) The precedence of operators and use of brackets is reflected in the abstract object representing the expression.

Abstract Syntax

(2) is-arithm-prefix-opr = is-PLUS ∨ is-MINUS

(3) is-real-prefix-expr = (<s-op:is-real-expr>,
<s-opr:is-arithm-prefix-opr>)

(4) is-intg-prefix-expr = (<s-op:is-intg-expr>,
<s-opr:is-arithm-prefix-opr>)

(5) is-real-infix-opr = is-PLUS ∨ is-MINUS ∨ is-MULT ∨ is-DIV ∨ is-POWER

(6) is-real-infix-expr-1 = (<s-op-1:is-real-expr>,
<s-op-2:is-real-expr>,
<s-opr:is-real-infix-opr>)

(7) is-real-infix-expr-2 = (<s-op-1:is-real-expr>,
<s-op-2:is-intg-expr>,
<s-opr:is-real-infix-opr>)

(8) is-real-infix-expr-3 = (<s-op-1:is-intg-expr>,
<s-op-2:is-real-expr>,
<s-opr:is-real-infix-opr>)

(9) is-real-infix-expr-4 = (<s-op-1:is-intg-expr>,
<s-op-2:(is-intg-expr & ¬is-non-neg-intg-const)>,
<s-opr:is-POWER>)

refs: is-non-neg-intg-const 2.5

(10) is-real-infix-expr-5 = (<s-op-1:is-intg-expr>,
<s-op-2:is-intg-expr>,
<s-opr:is-DIV>)

(11) is-real-infix-expr = is-real-infix-expr-1 ∨ is-real-infix-expr-2 ∨ is-real-infix-expr-3 ∨
is-real-infix-expr-4 ∨ is-real-infix-expr-5

3.3.1 Syntax

<adding operator> ::= + | -

<multiplying operator> ::= * | / | ÷

<primary> ::= <unsigned number> | <variable> | <function designator> | (<arithmetic expression>)

<factor> ::= <primary> | <factor> ↑ <primary>

<term> ::= <factor> | <term> <multiplying operator> <factor>

<simple arithmetic expression> ::= <term> | <adding operator> <term> |
<simple arithmetic expression> <adding operator> <term>

<if clause> ::= if <Boolean expression> then

<arithmetic expression> ::= <simple arithmetic expression> |
<if clause> <simple arithmetic expression> else <arithmetic expression

```
(12)  is-intg-infix-opr = is-PLUS v is-MINUS v is-MULT v is-INTGDIV v is-POWER

(13)  is-intg-infix-expr-1 = (<s-op-1:is-intg-expr>,
                              <s-op-2:is-intg-expr>,
                              <s-opr:(is-intg-infix-opr & ¬is-POWER)>)

(14)  is-intg-infix-expr-2 = (<s-op-1:is-intg-expr>,
                              <s-op-2:is-non-neg-intg-const>,
                              <s-opr:is-POWER>)

        refs: is-non-neg-intg-const 2.5

(15)  is-intg-infix-expr = is-intg-infix-expr-1 v is-intg-infix-expr-2

(16)  is-real-cond-expr-1 = (<s-decision:is-bool-expr>,
                             <s-then-expr:is-real-expr>,
                             <s-else-expr:is-real-expr>)

        refs: is-bool-expr 3.4

(17)  is-real-cond-expr-2 = (<s-decision:is-bool-expr>,
                             <s-then-expr:is-real-expr>,
                             <s-else-expr:is-intg-expr>)

        refs: is-bool-expr 3.4

(18)  is-real-cond-expr-3 = (<s-decision:is-bool-expr>,
                             <s-then-expr:is-intg-expr>,
                             <s-else-expr:is-real-expr>)

        refs: is-bool-expr 3.4

(19)  is-real-cond-expr = is-real-cond-expr-1 v is-real-cond-expr-2 v is-real-cond-expr-3

(20)  is-intg-cond-expr = (<s-decision:is-bool-expr>,
                           <s-then-expr:is-intg-expr>,
                           <s-else-expr:is-intg-expr>)

        refs: is-bool-expr 3.4
```


3.3.2 Examples

```
Primaries:
     7.394₁₀-8
     sum
     w[i+2,8]
     cos(y+z*3)
     (a-3/y+vu↑8)

Factors:
     omega
     sum↑cos(y+z*3)
     7.394₁₀-8↑w[i+2,8]↑(a-3/y+vu↑8)

Terms:
     U
     omega*sum↑cos(y+z*3)/7.394₁₀-8
          ↑w[i+2,8]↑(a-3/y+vu↑8)

Simple arithmetic expression:
     U-Yu+omega*sum↑cos(y+z*3)/7.394₁₀-8
          ↑w[i+2,8]↑(a-3/y+vu↑8)

Arithmetic expressions:
     w*u-Q(S+Cu)↑2
     if q>0 then S+3*Q/A else 2*S+3*q
     if a<0 then U+V else if a*b>17 then U/V else if k≠y
          then V/U else 0
     a*sin(omega*t)
     0.57₁₀12*a[N*(N-1)/2,0]
     (A*arctan(y)+Z)↑(7+Q)
     if q then n-1 else n
     if a<0 then A/B else if b=0 then B/A else z
```

(21) is-real-expr = is-real-op ∨ is-real-var ∨ is-real-funct-ref ∨ is-real-prefix-expr ∨
is-real-infix-expr ∨ is-real-cond-expr

refs: is-real-var 3.1, is-real-funct-ref 3.2

(22) is-intg-expr = is-intg-op ∨ is-intg-var ∨ is-intg-funct-ref ∨ is-intg-prefix-expr ∨
is-intg-infix-expr ∨ is-intg-cond-expr

refs: is-intg-var 3.1, is-intg-funct-ref 3.2

(23) is-arithm-expr = is-real-expr ∨ is-intg-expr

Auxiliary predicates

(24) is-arithm-prefix-expr = is-real-prefix-expr ∨ is-intg-prefix-expr

(25) is-arithm-infix-expr = is-real-infix-expr ∨ is-intg-infix-expr

(26) is-arithm-cond-expr = is-real-cond-expr ∨ is-intg-cond-expr

(27) is-cond-expr = is-arithm-cond-expr ∨ is-bool-cond-expr ∨ is-des-cond-expr

refs: is-bool-cond-expr 3.4, is-des-cond-expr 3.5

(28) is-arithm-infix-opr = is-real-infix-opr ∨ is-intg-infix-opr

Interpretation

```
(29)  apply-arithm-opr(e) =

          cases: e
          is-arithm-prefix-expr: arithm-prefix-opr(apply•s-op(e),s-opr(e))
          is-arithm-infix-expr: arithm-infix-opr(apply•s-op-1(e),apply•s-op-2(e),s-opr(e))
          is-arithm-cond-expr: cases: s-value•s-decision(e)
                               TRUE: cases: is-intg-expr•s-then-expr(e) & is-real-expr(e)
                                     TRUE: mk-op(REAL,s-value•apply(s-then-expr(e)))
                                     FALSE: apply•s-then-expr(e)
                               FALSE: cases: is-intg-expr•s-else-expr(e) & is-real-expr(e)
                                      TRUE: mk-op(REAL,s-value•apply(s-else-expr(e)))
                                      FALSE: apply•s-else-expr(e)

       refs: apply 3
       type: is-arithm-expr → is-arithm-op


(30)  arithm-prefix-opr(op,opr) =

          cases: s-type(op)
          INTG: cases: opr
                PLUS: op
                MINUS: mk-op(INTG, - s-value(op))
          REAL: mk-op(REAL,real-prefix-value(s-value(op),opr))

       type: is-arithm-op X is-arithm-prefix-opr → is-arithm-op


(31)  real-prefix-value(v,opr) =

       This function is implementation-defined.

       type: is-real-val X is-arithm-prefix-opr → is-real-val
```


3.3.3 Semantics

An arithmetic expression is a rule for computing a numerical value. In case of simple arithmetic expressions this value is obtained by executing the indicated arithmetic operations on the actual numerical values of the primaries of the expression, as explained in detail in section 3.3.4 below.

The actual numerical value of a primary is obvious in the case of numbers. For variables it is the current value (assigned last in the dynamic sense), and for function designators it is the value arising from the computing rules defining the procedure (cf. section 5.4.4 Values of function designators) when applied to the current values of the procedure parameters given in the expression. Finally, for arithmetic expressions enclosed in parentheses the value must through a recursive analysis be expressed in terms of the values of primaries of the other three kinds.

In the more general arithmetic expressions, which include *if* clauses, one out of several simple arithmetic expressions is selected on the basis of the actual values of the Boolean expressions (cf. section 3.4 Boolean expressions). This selection is made as follows: The Boolean expressions of the *if* clauses are evaluated one by one in sequence from left to right until one having the value *true* is found. The value of the arithmetic expression is then the value of the first arithmetic expression following this Boolean (the largest arithmetic expression found in this position is understood).

The construction:

else <simple arithmetic expression>

is equivalent to the construction:

else if true then <simple arithmetic expression>

COMMENT: No references within either branch of a conditional expression are made until the <Boolean expression> is referenced fully. When this is fully referenced it can be determined which branch is to be used, and no references in the other branch are made.

COMMENT: The type of a conditional expression depends on the types of both branches. The expression being of type *integer* only in the case when both branches are of type *integer*. Thus if the types of the branches are different, depending on

```
(32)  arithm-infix-opr(op-1,op-2,opr) =

        let: type¹ = s-type(op-1)
             type² = s-type(op-2)
             v¹ = s-value(op-1)
             v² = s-value(op-2)
        cases: (type¹,type²,opr)
        (is-arithm,INTG,POWER): intg-power-opr(op-1,op-2))
        (is-arithm,REAL,POWER): mk-op(REAL,real-power-value(v¹,v²))
        (INTG,INTG,is-intg-infix-opr): mk-op(INTG,intg-arithm-infix-value(v¹,v²,opr))
        (is-arithm,is-arithm,is-real-infix-opr):
                  mk-op(REAL,real-arithm-infix-value(v¹,v²,opr))

     refs: is-arithm 5.1
     type: is-arithm-op X is-arithm-op X is-arithm-infix-opr → is-arithm-op


(33)  intg-power-opr(op-1,op-2) =

        let: v¹ = intg-power-opr-val(op-1,s-value(op-2))
        cases: is-intg-op(op-1) & is-non-neg-intg-const(op-2)
        TRUE: mk-op(INTG,v¹)
        FALSE: mk-op(REAL,v¹)

     refs: is-non-neg-intg-const 2.5
     note: intg-op to power non-neg-intg-const yields an intg result.
     type: is-arithm-op X is-intg-op → is-arithm-op


(34)  intg-power-opr-val(op,i) =

        cases: i
        i>0: s-value•self-mult(op,i)
        i=0: cases: s-value(op) ≠ 0
             TRUE: mk-op(s-type(op),1)
             FALSE: error
        i<0: cases: s-value(op) ≠ 0
             TRUE: arithm-infix-opr(mk-op(REAL,1),self-mult(op,-i),DIV)
             FALSE: error

     error: If s-value(op) = 0 and i≤0.
     type: is-arithm-op X is-intg-val → is-arithm-val
```


which branch is evaluated, a conversion may be necessary. (see 3.3.4)

3.3.4 Operators and types

Apart from the Boolean expressions of if clauses, the constituents of simple arithmetic expressions must be of types real or integer (cf. section 5.1 Type declarations). The meaning of the basic operators and the types of the expressions to which they lead are given by a set of rules. However if the type of an arithmetic expression according to the rules cannot be determined without evaluating an expression or ascertaining the type or value of an actual parameter, it is real.

ECMA CHANGE

COMMENT: (See 3 and 3.3.4.3)

These rules are:

3.3.4.1 The operators +, -, and * have the conventional meaning (addition, subtraction, and multiplication). The type of the expression will be integer if both of the operands are of integer type, otherwise real.

3.3.4.2 The operations <term>/<factor> and <term> ÷ <factor> both denote division, to be understood as a multiplication of the term by the reciprocal of the factor with due regard to the rules of precedence (cf. section 3.3.5). Thus, for example

a/b * 7/(p - q) * v/s

means

((((a * (b↑-1)) * 7) *((p - q)↑-1)) * v) * (s↑-1)

The operator / is defined for all four combinations of types real and integer and will yield results of real type in any case. The operator ÷ is defined only for two operands both of type integer and will yield a result of type integer, mathematically defined as follows:

a ÷ b = sign (a/b) * entier (abs(a/b))
(cf. sections 3.2.4 and 3.2.5).

```
(35)  self-mult(op,i) =

          cases: i
          i>1: arithm-infix-opr(self-mult(op,i-1),op,MULT)
          i=1: op

      type: is-arithm-op X is-non-neg-intg-val → is-arithm-op

(36)  real-power-value(v-1,v-2) =

          cases: v-1
          v-1>0: real-arithm-infix-value(v-1,v-2,POWER)
          v-1=0: cases: v-2
                 v-2>0: 0
                 v-2≤0: error
          v-1<0: error

      error: If a zero value is raised to a negative or zero power, or a negative value is
             raised to any power.
      type: is-arithm-val X is-real-val → is-real-val

(37)  real-arithm-infix-value(v-1,v-2,opr) =

      This function is implementation defined.

      type: is-real-val X is-real-val X is-real-infix-opr → is-real-val

(38)  intg-arithm-infix-value(v-1,v-2,opr) =

          cases: opr
          PLUS: v-1 + v-2
          MINUS: v-1 - v-2
          MULT: v-1 * v-2
          INTGDIV: sign(v-1/v-2) * entier•abs(v-1/v-2)

      type: is-intg-val X is-intg-val X is-intg-infix-opr → is-intg-val
```


3.3.4.3 The operation <factor> ↑ <primary> denotes exponentiation, where the factor is the base and the primary is the exponent. Thus, for example

2 ↑ n ↑ k means (2↑n)↑k

while

2 ↑ (n ↑ m) means 2↑(n↑m)

Writing i for a number of integer type, r for a number of real type, and a for a number of either integer or real type, the result is given by the following rules:

a ↑ i
If i>0,a * a * ... * a(i times),of the same type as a.
If i=0,if a ≠ 0,1,of the same type as a, if a=0, undefined.
If i<0,if a ≠ 0, 1/(a * a * ... * a) (the denominator has -i factors), of type real, if a=0, undefined.

COMMENT: In the case where a is of type integer, then if i is not a constant, its value affects the value of the expression. In this case the expression is of type real. (see 3.3.4)

a ↑ r
If a>0,exp(r * ln(a)),of type real.
If a=0,if r >0, 0.0, of type real, if r<0, undefined.
If a<0, always undefined.

3.3.5 Precedence of operators

3.3.5.1 According to the syntax given in section 3.3.1 the following rules of precedence hold:

first: ↑
second: * / ÷
third: + -

3.3.5.2 The expression between a left parenthesis and the matching right parenthesis is evaluated by itself and this value is used in

subsequent calculations. Consequently the desired order of execution of operations within an expression can always be arranged by appropriate positioning of parentheses.

3.3.6 Arithmetics of real quantities

Numbers and variables of type real must be interpreted in the sense of numerical analysis, i.e. as entities defined inherently with only a finite accuracy. Similarly, the possibility of the occurrence of a finite deviation from the mathematically defined result in any arithmetic expression is explicitly understood. No exact arithmetic will be specified, however, and it is indeed understood that different hardware representations may evaluate arithmetic expressions differently. The control of the possible consequences of such differences must be carried out by the methods of numerical analysis. This control must be considered a part of the process to be described, and will therefore be expressed in terms of the language itself.

Translation

(1) The precedence of operators and use of brackets is reflected in the abstract object representing the expression.

Abstract Syntax

(2) is-bool-prefix-expr = (<s-opr:is-NOT>,<s-op:is-bool-expr>)

(3) is-bool-infix-opr = is-AND ∨ is-OR ∨ is-IMPL ∨ is-EQUIV

(4) is-bool-infix-expr = (<s-opr:is-bool-infix-opr>,
<s-op-1:is-bool-expr>,
<s-op-2:is-bool-expr>)

(5) is-bool-cond-expr = (<s-decision:is-bool-expr>,
<s-then-expr:is-bool-expr>,
<s-else-expr:is-bool-expr>)

(6) is-arithm-relat-opr = is-GT ∨ is-GE ∨ is-EQ ∨ is-LE ∨ is-LT ∨ is-NE

(7) is-arithm-relat-expr = (<s-opr:is-arithm-relat-opr>,
<s-op-1:is-arithm-expr>,
<s-op-2:is-arithm-expr>)

refs: is-arithm-expr 3.3

(8) is-bool-expr = is-bool-op ∨ is-bool-var ∨ is-bool-funct-ref ∨ is-bool-prefix-expr ∨
is-bool-infix-expr ∨ is-bool-cond-expr ∨ is-arithm-relat-expr

refs: is-bool-var 3.1, is-bool-funct-ref 3.2

3.4.1 Syntax

<relational operator> ::= < | ≤ | = | ≥ | > | ≠

<relation> ::= <simple arithmetic expression> <relational operator>
<simple arithmetic expression>

<Boolean primary> ::= <logical value> | <variable> | <function designator> | <relation> |
(<Boolean expression>)

<Boolean secondary> ::= <Boolean primary> | ¬ <Boolean primary>

<Boolean factor> ::= <Boolean secondary> | <Boolean factor> & <Boolean secondary>

<Boolean term> ::= <Boolean factor> | <Boolean term> ∨ <Boolean factor>

<implication> ::= <Boolean term> | <implication> ⊃ <Boolean term>

<simple Boolean> ::= <implication> | <simple Boolean> ≡ <implication>

<Boolean expression> ::= <simple Boolean> |
<if clause> <simple Boolean> else <Boolean expression>

3.4.2 Examples

```
x = -2
Y > V ∨ z<q
a+b > -5 & z-d>q↑2
p & q ∨ x≠y
g ≡ ¬a & b & ¬c ∨ d ∨ e ⊃ ¬f
if k<1 then s>w else h<c
if if if a then b else c then d else f then g else h<k
```

Interpretation

```
(9)   apply-bool-opr(e) =

          cases: e
          is-bool-prefix-expr: mk-op(BOOL, ¬ s-value•apply•s-op(e))
          is-bool-infix-expr: bool-infix-opr(apply•s-op-1(e),apply•s-op-2(e),s-opr(e))
          is-arithm-relat-expr: arithm-relat-opr(apply•s-op-1(e),apply•s-op-2(e),s-opr(e))
          is-bool-cond-expr: cases: s-value•s-decision(e)
                             TRUE: apply•s-then-expr(e)
                             FALSE: apply•s-else-expr(e)

      refs: apply 3
      type: is-bool-expr → is-bool-op


(10)  bool-infix-opr(op-1,op-2,opr) =

          mk-op(BOOL,bool-infix-value(s-value(op-1),s-value(op-2),opr))

      type: is-bool-op X is-bool-op X is-bool-infix-opr → is-bool-op


(11)  bool-infix-value(v-1,v-2,opr) =

          cases: opr
          AND  : v-1 & v-2
          OR   : v-1 ∨ v-2
          IMPL : v-1 ⊃ v-2
          EQUIV: v-1 ≡ v-2

      type: is-bool-val X is-bool-val X is-bool-infix-opr → is-bool-val


(12)  arithm-relat-opr(op-1,op-2,opr) =

          mk-op(BOOL,arithm-relat-value(s-value(op-1),s-value(op-2),opr))

      type: is-arithm-op X is-arithm-op X is-arithm-relat-opr → is-bool-op
```


3.4.3 Semantics

A Boolean expression is a rule for computing a logical value. The principles of evaluation are entirely analogous to those given for arithmetic expressions in section 3.3.3.

3.4.4 Types

Variables and function designators entered as Boolean primaries must be declared Boolean (cf. section 5.1 Type declarations and section 5.4.4 Values of function designators).

3.4.5 The operators

Relations take on the value true whenever the corresponding relation is satisfied for the expressions involved; otherwise false.

The meaning of the logical operators ¬(not), & (and), ∨ (or), ⊃ (implies), and ≡ (equivalent), is given by the following function table.

b1	false	false	true	true
b2	false	true	false	true
¬ b1	true	true	false	false
b1 & b2	false	false	false	true
b1 ∨ b2	false	true	true	true
b1 ⊃ b2	true	true	false	true
b1 ≡ b2	true	false	false	true

3.4.6 Precedence of operators

The sequence of operations within one expression is generally from left to right, with the following additional rules:

(13) arithm-relat-value(v-1,v-2,opr) =

cases: opr
GT: v-1 > v-2
GE: v-1 ≥ v-2
EQ: v-1 = v-2
LE: v-1 ≤ v-2
LT: v-1 < v-2
NE: v-1 ≠ v-2

type: is-arithm-val X is-arithm-val X is-relat-opr → is-bool-val

3.5 DESIGNATIONAL EXPRESSIONS

Translation

(1) is-path(path) & is-switch-des•path(prog) ⊃ (is-switch-desc•desc(s-id•path,prog) ∨ is-SWITCH•desc(s-id•path,prog))

refs: is-switch-desc 5.3

(2) is-path(path) & is-label-const(path(prog)) ⊃ is-label-desc(desc(s-id•path,prog))

refs: is-label-desc 5

(3) is-path(path) & is-label-var(path(prog)) ⊃ is-LABEL(desc(s-id•path,prog))

note: This only applies to the translation of label parameters.

Abstract Syntax

(4) is-switch-des = (<s-id:is-id>,
<s-subscr-list:(<elem(1):is-arithm-expr>)>,
<s-type:is-SWITCH>)

refs: is-id 2.4, is-arithm-expr 3.3

3.4.6.1 According to the syntax given in section 3.4.1 the following rules of precedence hold:

first:arithmetic expressions according to section 3.3.5.
second: < ≤ = ≥ > ≠
third: ¬
fourth: &
fifth: ∨
sixth: ⊃
seventh: ≡

3.4.6.2 The use of parentheses will be interpreted in the sense given in section 3.3.5.2.

3.5 DESIGNATIONAL EXPRESSIONS

3.5.1 Syntax

<label> ::= <identifier>

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<switch identifier> ::= <identifier>

```
(5)   is-des-cond-expr = (<s-decision:is-bool-expr>,
                          <s-then-expr:is-des-expr>,
                          <s-else-expr:is-des-expr>)

        refs: is-bool-expr 3.4

(6)   is-label-var = (<s-id:is-id>,
                      <s-type:is-LABEL>)

        refs: is-id 2.4

(7)   is-label-const = (<s-type:is-LABEL>,
                        <s-value:is-id>,
                        <s-const:is-CONST>)

        refs: is-id 2.4

(8)   is-des-expr = is-label-op ∨ is-label-var ∨ is-switch-des ∨ is-des-cond-expr
```

Interpretation

```
(9)   apply-des-opr(e) =

          cases: s-value•s-decision(e)
          TRUE: apply•s-then-expr(e)
          FALSE: apply•s-else-expr(e)

      refs: apply 3
      type: is-des-cond-expr → is-label-op
```



```
<switch designator> ::= <switch identifier> [ <subscript expression> ]

<simple designational expression> ::= <label> | <switch designator> |
                                      ( <designational expression> )

<designational expression> ::= <simple designational expression> |
                               <if clause> <simple designational expression> else
                                                    <designational expression>
```

3.5.2 Examples

```
17
     NON ECMA LANGUAGE
p9
Choose[n-1]
Town[if y<0 then N else N+1]
if Ab<c then 17 else q[if w≤0 then 2 else n]
     NON ECMA LANGUAGE
```

3.5.3 Semantics

A designational expression is a rule for obtaining a label of a statement (cf. section 4 Statements). Again, the principle of the evaluation is entirely analogous to that of arithmetic expressions (section 3.3.3). In the general case the Boolean expressions of the if clauses will select a simple designational expression. If this is a label the desired result is already found. A switch designator refers to the corresponding switch declaration (cf. section 5.3 Switch declarations) and by the actual numerical value of its subscript expression selects one of the designational expressions listed in the switch declaration by counting these from left to right. Since the designational expression thus selected may again by a switch designator this evaluation is obviously a recursive process.

3.5.4 The subscript expression

The evaluation of the subscript expression is analogous to that of subscripted variables (cf. section 3.1.4.2). The value of a switch designator is defined only if the subscript expression assumes one of the positive values 1,2,3,...,n, where n is the number of entries in the switch list.

3.5.5

Deleted.

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Translation

(1) The <labels> at the head of a <basic statement> etc. are collected into a set.

(2) The Translator rejects any <program> in which errors of duplication would be hidden by (1) (e.g. lab:lab:x:=1;).

(3) local(id,t) =

(∃sel)(id∈s-label-pt•sel(t) & ¬(∃sel-1,sel-2)(sel=sel-2•sel-1 & is-block•sel-1(t)))

refs: is-block 4.1
note: It is implicit in the above definition that labels within procedure declarations are omitted because no selector will yield components of the declaration set.
type: is-id X is-text → is-bool-val

(4) make-st-sel(id,t) =

(i sel)(id∈s-label-pt•sel(t) & ¬(∃sel-1,sel-2)(sel=sel-2•sel-1 & is-block•sel-1(t)))

refs: is-block 4.1
note: Only used if local(id,t).
type: is-id X is-text → is-lab-sel

Abstract syntax

(5) is-unlab-st = is-comp-st ∨ is-block ∨ is-assign-st ∨ is-goto-st ∨ is-dummy-st ∨ is-cond-st ∨ is-for-st ∨ is-proc-st

refs: is-comp-st 4.1, is-block 4.1, is-assign-st 4.2, is-goto-st 4.3, is-dummy-st 4.4, is-cond-st 4.5, is-for-st 4.6, is-proc-st 4.7

(6) is-st = (<s-label-pt:is-id-set>, <s-st-pt:is-unlab-st>)

refs: is-id 2.4

The units of operation within the language are called statements. They will normally be executed consecutively as written. However, this sequence of operations may be broken by goto statements, which define their successor explicitly, and shortened by conditional statements, which may cause certain statements to be skipped.

In order to make it possible to define a specific dynamic succession, statements may be provided with labels.

Since sequences of statements may be grouped together into compound statements and blocks, the definition of statement must necessarily be recursive. Also since declarations, described in section 5, enter fundamentally into the syntactic structure, the syntactic definition of statements must suppose declarations to be already defined.

Interpretation

```
(7)   int-st(st,dn,vl) =

          cases: int-unlab-st(s-st-pt(st),dn,vl)
          (vl¹,Ω): (vl¹,Ω)
          (vl¹,lab¹): cases: lab¹
                      local(lab¹,st): cue-int-st(make-st-sel(lab¹,st),st,dn,vl¹)
                      ¬local(lab¹,st): (vl¹,lab¹)

       type: is-st X is-dn X is-vl → is-vl X is-abn


(8)   int-unlab-st(t,dn,vl) =

          cases: t
          is-comp-st: int-st-list(t,1,dn,vl)
          is-block: int-block(t,dn,vl)
          is-assign-st: int-assign-st(t,dn,vl)
          is-goto-st: int-goto-st(t,dn,vl)
          is-dummy-st: (vl,Ω)
          is-cond-st: int-cond-st(t,dn,vl)
          is-for-st: int-for-st(t,dn,vl)
          is-proc-st: int-proc-st(t,dn,vl)

       refs: is-comp-st 4.1, int-st-list 4.1, is-block 4.1, int-block 4.1, is-assign-st 4.2,
             int-assign-st 4.2, is-goto-st 4.3, int-goto-st 4.3, is-dummy-st 4.4, is-cond-st 4.5,
             int-cond-st 4.5, is-for-st 4.6, int-for-st 4.6, is-proc-st 4.7, int-proc-st 4.7
       type: is-unlab-st X is-dn X is-vl → is-vl X is-abn


(9)   cue-int-st(targ-sel,st,dn,vl) =

          cases: targ-sel
          I: int-st(st,dn,vl)
          ¬is-I: cases: cue-int-unlab-st(rest-pt(targ-sel),s-st-pt(st),dn,vl)
                 (vl¹,Ω): (vl¹,Ω)
                 (vl¹,lab¹): cases: lab¹
                             local(lab¹,st): cue-int-st(make-st-sel(lab¹,st),st,dn,vl¹)
                             ¬local(lab¹,st): (vl¹,lab¹)

       type: is-lab-sel X is-st X is-dn X is-vl → is-vl X is-abn
```

```
(10)  cue-int-unlab-st(targ-sel,t,dn,vl) =

          cases: t
          is-comp-st: let: i¹ = (i i)(main-pt(targ-sel) = elem(i))
                      cue-int-st-list(rest-pt(targ-sel),t,i¹,dn,vl)
          is-cond-st: cue-int-st(rest-pt(targ-sel),main-pt(targ-sel)(t),dn,vl)
          is-for-st: error

       refs: is-comp-st 4.1, cue-int-st-list 4.1, is-cond-st 4.5, is-for-st 4.6
       error: goto into for not allowed (see A.R. 4.6.6).
       type: is-lab-sel X is-unlab-st X is-dn X is-vl → is-vl X is-abn
```


Translation

(1) All <programs> are surrounded by an embracing block which introduces all of the standard functions (see AR 3.2.4). SIGN must be included since it is used in interpretation as though referenced in the text.

(2) All <procedure bodies> which are not <code> are converted to blocks.

```
(3)  intr-ids(t) =

          {id | (∃d)(d ∈ s-decl-pt(t) & id ∈ s-id-set(d))} ∪ local-labs(t)

       type: is-block → is-id-set

(4)  local-labs(t) =

          {id | local(id,t)}

       refs: local 4
       type: is-block → is-id-set
```


4.1 COMPOUND STATEMENTS AND BLOCKS

```
(5)   desc-block(id,path-el•path,t) =

          cases: id
          (∃decl)(decl ∈ s-decl-pt•path-el•path(t) &  id ∈ s-id-set(decl)):
               let: decl' = (ι decl)(decl∈s-decl-pt•path-el•path(t) & id ∈ s-id-set(decl))
               s-desc(decl')
          id ∈ local-labs(path-el•path(t)): μ0(<s-type:LABEL>)
          T: desc-1(id,path,t)

      type: is-id X is-path X is-program → (is-specifier ∨ is-desc ∨ is-label-desc)
```

Abstract syntax

```
(6)   is-comp-st = is-st-list

      refs: is-st 4

(7)   is-block = (<s-decl-pt:is-decl-set>,
                  <s-st-list:is-st-list>)

      refs: is-decl 5, is-st 4

(8)   is-program = is-block
```


4.1.1 Syntax

<unlabelled basic statement> ::= <assignment statement> | <goto statement> | <dummy statement> | <procedure statement>

<basic statement> ::= <unlabelled basic statement> | <label> : <basic statement>

<unconditional statement> ::= <basic statement> | <compound statement> | <block>

<statement> ::= <unconditional statement> | <conditional statement> | <for statement>

<compound tail> ::= <statement> end | <statement> ; <compound tail>

<block head> ::= begin <declaration> | <block head> ; <declaration>

<unlabelled compound> ::= begin <compound tail>

<unlabelled block> ::= <block head> ; <compound tail>

<compound statement> ::= <unlabelled compound> | <label> : <compound statement>

<block> ::= <unlabelled block> | <label> : <block>

<program> ::= <block> | <compound statement>

This syntax may be illustrated as follows: Denoting arbitrary statements, declarations, and labels, by letters S, D, and L, respectively, the basic syntactic units take the forms:

Compound statement:

L:L:...begin S;S;...S;S end

Interpretation

```
(9)    int-program(t) =

           cases: int-block(t,{ },{ })
           ({ },Ω): EMPTY

       note: Only possible case.
       type: is-program → EMPTY


(10)   int-block(t,dn,vl) =

           cases: eval-array-decls(s-decl-pt(t),dn,vl)
           (decl-set¹,vl¹,Ω):
                let: pr-set¹ = mk-pairs(intr-ids(t),t,dn)
                     t¹ = change-block(μ(t:<s-decl-pt:decl-set¹>),pr-set¹)
```


Block:

```
L:L:...begin D;D;...D;S;S;...S;S end
```

It should be kept in mind that each of the statements S may again be a complete compound statement or block.

4.1.2 Examples

```
Basic statements:
     a := p+q
     goto Naples
     START:CONTINUE:W:=7.993

Compound statement:
     begin x := 0; for y:=1 step 1 until n do x:= x+A[y];
          if x>q then goto STOP else if x>w-2 then goto S;
          Aw:St:W:= x+bob end

Block:
     Q: begin integer i,k; real w;
          for i:=1 step 1 until m do
          for k:=i+1 step 1 until m do
          begin w:=A[i,k];A[i,k]:=A[k,i];
               A[k,i]:=w
               end for i and k
          end block Q
```

4.1.3 Semantics

Every block automatically introduces a new level of nomenclature. This is realized as follows: Any identifier occurring within the block may through a suitable declaration (cf. section 5 Declarations) be specified to be local to the block in question. This means (a) that the entity represented by this identifier inside the block has no existence outside it, and (b) that any entity represented by this identifier outside the block is completely inaccessible inside the block.

Identifiers (except those representing labels) occurring within a block and not being declared to this block will be non-local to it, i.e. will represent the same entity inside the block and in the level immediately outside it. A label separated by a colon from a statement, i.e. labelling that statement, behaves as though declared in the head of the smallest embracing block, i.e. the smallest block whose brackets begin and end enclose that statement. In this context a procedure body must be considered as if it were enclosed by begin and end and treated as a block.

```
                labs¹ = μ₀(<s-id-set:{id | id∈local-labs(t¹)}>,
                           <s-desc:μ₀(<s-type:is-LABEL>)>)
                dn¹ = augment-dn(s-decl-pt(t¹) ∪ {labs¹},dn)
                vl² = mod-set(vl¹,{<id,{}>|decl∈s-decl-pt(t¹) &
                                    is-array-desc∘s-desc(decl) & id ∈ s-id-set(decl)})
                (vl³,abn³) = int-block-body(s-st-list(t¹),dn¹,vl²)
                vl⁴ = epilogue(seconds(pr-set¹),vl³)
            (vl⁴,abn³)
        (decl-set¹,vl¹,lab¹): (vl¹,lab¹)
```

refs: eval-array-decls 5.2, change-block 4.7, augment-dn 5, is-array-desc 5.2
note: Array bounds are evaluated before the new dn is installed, no local refs are possible, nor are local gotos (see A.R. 5.2.4.2).
type: is-block X is-dn X is-vl → is-vl X is-abn

(11) mk-pairs(id-set,t,dn) =

```
    let: used-set¹ = all-intrs(t) ∪ {id | pden ∈ seconds(dn) & is-proc-den(pden) &
                                          id ∈ all-intrs(pden)}
         avoid-set¹ = id-set ∪ used-set¹
    construct-pairs(id-set,used-set¹,avoid-set¹)
```

type: is-id-set X (is-block ∨ is-proc-den) X is-dn → is-idpr-set

(12) construct-pairs(id-set,us-set,av-set) =

```
    cases: id-set
    {}: { }
    ¬is-{}: for some id ∈ id-set
            let: id¹ = cases: id ∈ us-set
                       FALSE: id
                       TRUE: for some id² ∈ {id | is-id(id) & ¬(id ∈ av-set)}
                             id²
            {<id,id¹>} ∪ construct-pairs(id-set - {id},us-set,av-set ∪ {id¹})
```

refs: is-id 2.4
note: The arbitrary order does not affect the result.
No name is chosen which either has been used or is bound in a piece of text which can become active.
type: is-id-set X is-id-set X is-id-set → is-idpr-set

Since a statement of a block may again itself be a block the concepts local and non-local to a block must be understood recursively. Thus an identifier, which is non-local to a block A, may or may not be non-local to the block B in which A is one statement.

(13) all-intrs(t) =

```
{id | (∃path)(path≠I & (is-block(path(t)) & (id ∈ intr-ids(path(t)) ∨
      is-proc-desc(path(t)) & (∃i)(elem(i)•s-form-par-list(path(t))=id))) }
```

refs: is-proc-desc 5.4
note: This function collects all identifiers bound in nested blocks or procedure declarations. (Not those of the argument text.)
type: is-text → is-id-set

(14) change-block(t,pr-set) =

```
μ0(<s-decl-pt:{change-text(d,pr-set) | d∈s-decl-pt(t)}>,
   <s-st-list:change-text(s-st-list(t),pr-set)>)
```

refs: change-text 4.7
note: Split in this way in order to change the outer block with a non-deleted set. Always produces an object satisfying is-block because id-prs used.
type: is-block X is-id-pr-set → is-block

(15) int-block-body(t,dn,vl) =

```
int-st-list(t,1,dn,vl)
```

type: is-st-list X is-dn X is-vl → is-vl X is-abn

(16) int-st-list(t,i,dn,vl) =

```
cases: i
i > length(t): (vl,Ω)
i ≤ length(t):
     cases: int-st(elem(i,t),dn,vl)
     (vl¹,Ω): int-st-list(t,i+1,dn,vl¹)
     (vl¹,lab¹): cases: lab¹
                 local(lab¹,t): let: ext = make-st-sel(lab¹,t)
                                     i¹ = (i j)(elem(j) = main-pt(ext))
                                cue-int-st-list(rest-pt(ext),t,i¹,dn,vl¹)
                 ¬local(lab¹,t): (vl¹,lab¹)
```

refs: int-st 4, local 4, make-st-sel 4
type: is-st-list X is-intg-val X is-dn X is-vl → is-vl X is-abn

```
(17)   cue-int-st-list(targ-sel,t,i,dn,vl) =

           cases: cue-int-st(targ-sel,elem(i,t),dn,vl)
           (vl¹,Ω): int-st-list(t,i+1,dn,vl¹)
           (vl¹,lab¹): cases: lab¹
                       local(lab¹,t): let: ext = make-st-sel(lab¹,t)
                                           i¹ = (i j)(elem(j) = main-pt(ext))
                                      cue-int-st-list(rest-pt(ext),t,i¹,dn,vl¹)
                       ¬local(lab¹,t): (vl¹,lab¹)

       refs: cue-int-st 4, local 4, make-st-sel 4
       type: is-lab-sel X is-st-list X is-intg-val X is-dn X is-vl → is-vl X is-abn


(18)   epilogue(id-set,vl) =

           del-set(vl,id-set)

       type: is-id-set X is-vl → is-vl
```

Translation

A procedure identifier can only appear on the left of an assignment statement if it is a type procedure and the assignment statement is within the body of the procedure itself. In such cases the type of the procedure is noted in the abstract text.

(1) is-path(path) & is-real-activated-fn(path(prog)) ⊃
(∃ path-1,path-2)(is-proc-desc•path-1(prog) & path = path-2•path-1 &
path-1(prog) = desc(s-id•path,prog) &
is-REAL-PROC•s-type•desc(s-id•path,prog))

refs: is-proc-desc 5.4

(2) is-path(path) & is-intg-activated-fn(path(prog)) ⊃
(∃ path-1,path-2)(is-proc-desc•path-1(prog) & path = path-2•path-1 &
path-1(prog) = desc(s-id•path,prog) &
is-INTG-PROC•s-type•desc(s-id•path,prog))

refs: is-proc-desc 5.4

(3) is-path(path) & is-bool-activated-fn(path(prog)) ⊃
(∃ path-1,path-2)(is-proc-desc•path-1(prog) & path = path-2•path-1 &
path-1(prog) = desc(s-id•path,prog) &
is-BOOL-PROC•s-type•desc(s-id•path,prog))

refs: is-proc-desc 5.4

Abstract syntax

(4) is-real-activated-fn = (<s-id:is-id>,
<s-type:is-REAL-PROC>)

refs: is-id 2.4

(5) is-real-lp = is-real-var ∨ is-real-activated-fn

refs: is-real-var 3.1

4.2.1 Syntax

<left part> ::= <variable> := | <procedure identifier> :=

<left part list> ::= <left part> | <left part list> <left part>

<assignment statement> ::= <left part list> <arithmetic expression> |
<left part list> <Boolean expression>

(6) is-intg-activated-fn = (<s-id:is-id>,
<s-type:is-INTG-PROC>)

refs: is-id 2.4

(7) is-intg-lp = is-intg-var ∨ is-intg-activated-fn

refs: is-intg-var 3.1

(8) is-bool-activated-fn = (<s-id:is-id>,
<s-type:is-BOOL-PROC>)

refs: is-id 2.4

(9) is-bool-lp = is-bool-var ∨ is-bool-activated-fn

refs: is-bool-var 3.1

(10) is-real-assign-st = (<s-lp:(is-real-lp-list & ¬is-<>)>,
<s-rp:is-arithm-expr>)

refs: is-arithm-expr 3.3

(11) is-intg-assign-st = (<s-lp:(is-intg-lp-list & ¬is-<>)>,
<s-rp:is-arithm-expr>)

refs: is-arithm-expr 3.3

(12) is-bool-assign-st = (<s-lp:(is-bool-lp-list & ¬is-<>)>,
<s-rp:is-bool-expr>)

refs: is-bool-expr 3.4

(13) is-assign-st = is-intg-assign-st ∨ is-real-assign-st ∨ is-bool-assign-st

4.2.2 Examples

```
s := p[0] := n := n+1+s
n := n+1
A := B/C-v-q*S
S[v,k+2] := 3-arctan(s*zeta)
V := Q>Y & Z
```

Auxiliary predicates

(14) is-activated-fn = is-real-activated-fn ∨ is-intg-activated-fn ∨ is-bool-activated-fn

(15) is-lp = is-real-lp ∨ is-intg-lp ∨ is-bool-lp

Interpretation

```
(16)  int-assign-st(t,dn,vl) =

          cases: eval-lp-list(s-lp(t),dn,vl)
          (lp-list¹,vl¹,Ω):
               cases: eval-expr(s-rp(t),dn,vl¹)
               (op²,vl²,Ω): (change-lp-vars(lp-list¹,op²,dn,vl²),Ω)
               (op²,vl²,lab²): (vl²,lab²)
          (lp-list¹,vl¹,lab¹): (vl¹,lab¹)

        ref: eval-expr 3
        note: All left parts now satisfy is-var (cf. insert-ret 3.2).
        type: is-assign-st X is-dn X is-vl → is-vl X is-abn


(17)  eval-lp-list(lp-list,dn,vl) =

          cases: lp-list
          <>: (<>,vl,Ω)
          ¬is-<>:
               cases: eval-lp(head(lp-list),dn,vl)
               (lp¹,vl¹,Ω):
                    cases: eval-lp-list(tail(lp-list),dn,vl¹)
                    (lp-list²,vl²,Ω): (<lp¹>^lp-list²,vl²,Ω)
                    (lp-list²,vl²,lab²): (Ω,vl²,lab²)
               (lp¹,vl¹,lab¹): (Ω,vl¹,lab¹)

        type: is-var-list X is-dn X is-vl → is-opt-var-list X is-vl X is-abn
```


4.2.3 Semantics

Assignment statements serve for assigning the value of an expression to one or several variables or procedure identifiers. Assignment to a procedure identifier may only occur within the body of a procedure defining the value of a function designator (cf. section 5.4.4). The process will in the general case be understood to take place in three steps as follows:

4.2.3.1 Any subscript expressions occurring in the left part variables are evaluated in sequence from left to right.

COMMENT: This is taken to mean that all subscript expressions of variables of the left part list are evaluated in sequence from left to right. (cf. 3)

4.2.3.2 The expression of the statement is evaluated.

4.2.3.3 The value of the expression is assigned to all the left part variables, with any subscript expressions having values as evaluated in step 4.2.3.1.

4.2.4 Types

The type associated with all variables and procedure identifiers of a left part list must be the same. If this type is Boolean, the expression must likewise be Boolean. If the type is real or integer, the expression must be arithmetic. If the type of the arithmetic expression differs from that associated with the variables and procedure identifiers, appropriate transfer functions are understood to be automatically invoked. For transfer from real to integer type the transfer function is understood to yield a result equivalent to

```
entier(E + 0.5)
```

where E is the value of the expression. The type associated with a procedure identifier is given by the declarator which appears as the first symbol of the corresponding procedure declaration (cf. section 5.4.4).

```
(18)   eval-lp(lp,dn,vl) =

           cases: lp
           is-simple-var: (lp,vl,Ω)
           is-subscr-var:
                cases: eval-subs(s-subscr-list(lp),dn,vl)
                (op-list¹,vl¹,Ω):
                     let: e-sub-list = convert-subs(s-bounds•s(s-id(lp),dn),op-list¹)
                          var¹ = μ(lp;<s-subscr-list:e-sub-list>)
                     (var¹,vl¹,Ω)
                (op-list¹,vl¹,lab¹): (Ω,vl¹,lab¹)

       refs: is-simple-var 3.1, is-subscr-var 3.1, convert-subs 3.1
       type: is-var X is-dn X is-vl → is-opt-var X is-vl X is-abn


(19)   eval-subs(sub-list,dn,vl) =

           cases: sub-list
           <>: <>
           ¬is-<>:
                cases: eval-expr(head(sub-list),dn,vl)
                (e-hd,vl¹,Ω):
                     cases: eval-subs(tail(sub-list),dn,vl¹)
                     (e-tl,vl²,Ω): (<e-hd>ᵐe-tl,vl²,Ω)
                     (e-tl,vl²,lab²): (Ω,vl²,lab²)
                (e-hd,vl¹,lab¹): (Ω,vl¹,lab¹)

       refs: eval-expr 3
       type: is-arithm-expr-list X is-dn X is-vl → is-opt-op-list X is-vl X is-abn


(20)   change-lp-vars(lp-list,op,dn,vl) =

           let: v¹ = cases: lp-list
                     is-real-lp-list: convert(REAL,op)
                     is-intg-lp-list: convert(INTG,op)
                     is-bool-lp-list: op
                assign-to-lp-list(lp-list,v¹,vl)

       type: is-var-list X is-op X is-dn X is-vl → is-vl
```

```
(21)   convert(type,op) =

           cases: (type,s-type(op))
           (REAL,INTG): s-value(op)
           (INTG,REAL): entier(s-value(op) + 0.5)
           T:           s-value(op)

       type: is-arithm X is-arithm-op → is-arithm-val


(22)   assign-to-lp-list(lp-list,val,vl) =

           cases: lp-list
           <>: vl
           ¬is-<>: let: vl¹ = assign(head(lp-list),val,vl)
                   assign-to-lp-list(tail(lp-list),val,vl¹)

       type: is-var-list X is-simple-val X is-vl → is-vl


(23)   assign(lp,v,vl) =

           cases: lp
           is-simple-var: mod-set(vl,{<s-id(lp),v>})
           is-subscr-var: mod-set(vl,{<s-id(lp),mod-set(s(s-id(lp),vl),
                                                        {<s-subscr-list(lp),v>})>})

       refs: is-simple-var 3.1, is-subscr-var 3.1
       type: is-var X is-simple-val X is-vl → is-vl
```

4.3 GOTO STATEMENTS

Abstract syntax

```
(1)    is-goto-st = (<s-des-expr:is-des-expr>)

       refs: is-des-expr 3.5
```


4.3 GO TO STATEMENTS

4.3.1 Syntax

<go to statement> ::= goto <designational expression>

Interpretation

(2) int-goto-st(t,dn,vl) =

```
        cases: eval-expr(s-des-expr(t),dn,vl)
        (lab-den,vl¹,Ω): (vl¹,s-value(lab-den))
        (lab-den,vl¹,lab¹): (vl¹,lab¹)

    refs: eval-expr 3
    note: Switch designators whose value is undefined give error in
          eval-expr (cf. A.R. 4.3.5).
    type: is-goto-st X is-dn X is-vl → is-vl X is-abn
```

4.4 DUMMY STATEMENTS

Translation

(1) The elementary object DUMMY is inserted in place of the <dummy statement>.

4.3.2 Examples

```
goto 8
     NON ECMA LANGUAGE
goto exit[n+1]
goto Town[if y<0 then N else N+1]
goto if Ab<c then 17 else q[if w<0 then 2 else n]
     NON ECMA LANGUAGE
```

4.3.3 Semantics

A goto statement interrupts the normal sequence of operations, defined by the write-up of statements, by defining its successor explicitly by the value of a designational expression. Thus the next statement to be executed will be the one having this value as its label.

4.3.4 Restriction

Since labels are inherently local, no goto statement can lead from outside into a block. A goto statement may, however, lead from outside into a compound statement.

4.3.5 Go to an undefined switch designator

A goto statement is undefined if the designational expression is a switch designator whose value is undefined.

ECMA CHANGE

4.4 DUMMY STATEMENTS

Abstract syntax

(2) is-dummy-st = is-DUMMY

Interpretation

(3) see int-unlab-st :- is-dummy-st

refs: int-unlab-st 4

4.5 CONDITIONAL STATEMENTS

Translation

(1) A dummy-st is inserted as the s-else-st if none is present in the <conditional statement>.

Abstract syntax

(2) is-cond-st = (<s-decision:is-bool-expr>,
<s-then-st:is-st>,
<s-else-st:is-st>)

refs: is-bool-expr 3.4, is-st 4
note: The concrete syntax is such that: ¬is-cond-st(s-st-pt(s-then-st(t))).

4.4.1 Syntax

<dummy statement> :: = <empty>

4.4.2 Examples

L:
begin...;John:end

4.4.3 Semantics

A dummy statement executes no operation. It may serve to place a label.

4.5 CONDITIONAL STATEMENTS

4.5.1 Syntax

<if clause> ::= if <Boolean expression> then

<unconditional statement> ::= <basic statement> | <compound statement> | <block>

<if statement> ::= <if clause> <unconditional statement>

<conditional statement> ::= <if statement> | <if statement> else <statement> |
<if clause> <for statement> | <label> : <conditional statement>

4.5.2 Examples

```
if x>0 then n:=n+1
if v>u then V:q:=n+m else goto R
if s<0 v P<Q then AA:begin if q<v then a:=v/s
     else y:=2*a end
     else if v>s then a:=v-q
     else if v>s-1 then goto S
```

Interpretation

```
(3)   int-cond-st(t,dn,vl) =

          cases:eval-expr(s-decision(t),dn,vl)
          (bool-op¹,vl¹,Ω): cases: s-value(bool-op¹)
                            TRUE: int-st(s-then-st(t),dn,vl¹)
                            FALSE: int-st(s-else-st(t),dn,vl¹)
          (bool-op¹,vl¹,lab¹): (vl¹,lab¹)

      refs: eval-expr 3, int-st 4
      type: is-cond-st X is-dn X is-vl → is-vl X is-abn
```


4.5.3 Semantics

Conditional statements cause certain statements to be executed or skipped depending on the running values of specified Boolean expressions.

4.5.3.1 If statement. The unconditional statement of an if statement will be executed if the Boolean expression of the if clause is true. Otherwise it will be skipped and the operation will be continued with the next statement.

4.5.3.2 Conditional statement. According to the syntax two different forms of conditional statements are possible. These may be illustrated as follows:

if B1 then S1 else if B2 then S2 else S3;S4

and

if B1 then S1 else if B2 then S2 else if B3 then S3;S4

Here B1 to B3 are Boolean expressions, while S1 to S3 are unconditional statements. S4 is the statement following the complete conditional statement.

The execution of a conditional statement may be described as follows: The Boolean expressions of the if clauses are evaluated one after the other in sequence from left to right until one yielding the value true is found. Then the unconditional statement following this Boolean is executed. Unless this statement defines its successor explicitly the next statement to be executed will be S4, the statement following the complete conditional statement. Thus the effect of the delimiter else may be described by saying that it defines the successor of the statement it follows to be the statement following the complete conditional statement.

The construction

else <unconditional statement>

is equivalent to

else if true then <unconditional statement>

If none of the Boolean expressions of the if clauses is true, the effect of the whole conditional statement will be equivalent to that of a dummy statement.

4.6 FOR STATEMENTS

Abstract syntax

(1) is-while-elem = (<s-init-expr:is-arithm-expr>,
<s-while-expr:is-bool-expr>)

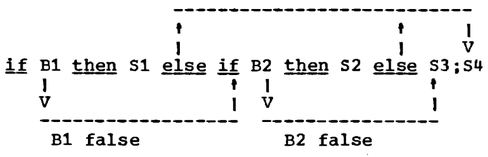
refs: is-arithm-expr 3.3, is-bool-expr 3.4

(2) is-step-until-elem = (<s-init-expr:is-arithm-expr>,
<s-step-expr:is-arithm-expr>,
<s-until-expr:is-arithm-expr>)

refs: is-arithm-expr 3.3

COMMENT: This equivalence is taken to be an indication of the sequencing and not to require the reversal of any side effects of the evaluation of the boolean expression.

For further explanation the following picture may be useful:



4.5.4 Go to into a conditional statement

The effect of a goto statement leading into a conditional statement follows directly from the above explanation of the effect of else.

4.6 FOR STATEMENTS

4.6.1 Syntax

<for list element> ::= <arithmetic expression> |
<arithmetic expression> step <arithmetic expression> until
<arithmetic expression> |
<arithmetic expression> while <Boolean expression>

<for list> ::= <for list element> | <for list> , <for list element>

<for clause> ::= for <variable> := <for list> do

<for statement> ::= <for clause> <statement> | <label> : <for statement>

(3) is-for-elem = is-arithm-expr ∨ is-while-elem ∨ is-step-until-elem

refs: is-arithm-expr 3.3

(4) is-for-st = (<s-contr-var:is-arithm-var>,
<s-for-list:(is-for-elem-list & ¬is-<>)>,
<s-st:is-st>)

refs: is-arithm-var 3.1, is-st 4

Interpretation

(5) int-for-st(t,dn,vl) =

cases: eval-lp(s-contr-var(t),dn,vl)
(cvar,vl¹,Ω): iterate-for-list(cvar,s-for-list(t),s-st(t),dn,vl¹)
(cvar,vl¹,lab¹): (vl¹,lab¹)

refs: eval-lp 4.2
note: goto from evaluation of a for list is considered as external to a for statement.
The control variable is evaluated once only.
type: is-for-st X is-dn X is-vl → is-vl X is-abn

(6) iterate-for-list(cvar,for-list,t,dn,vl) =

cases: for-list
<>: cases: cvar
is-simple-var: (del-set(vl,s-id(cvar)),Ω)
is-subscr-var: (mod-set(vl,{<s-id(cvar),del-set(s(s-id(cvar),vl),
s-subscr-list(cvar))>}),Ω)
¬is-<>: cases: iterate-for(cvar,head(for-list),t,dn,vl)
(vl¹,Ω): iterate-for-list(cvar,tail(for-list),t,dn,vl¹)
(vl¹,lab¹): (vl¹,lab¹)

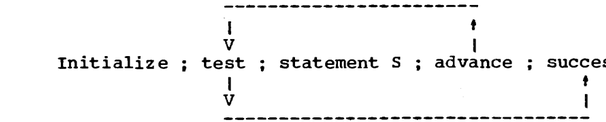
refs: is-simple-var 3.1, is-subscr-var 3.1
type: is-arithm-var X is-for-elem-list X is-st X is-dn X is-vl → is-vl X is-abn

4.6.2 Examples

```
for q:=1 step s until n do A[q]:=B[q]
for k:=1, V1*2 while V1<N do
    for j:=I+G, L, 1 step 1 until N, C+D do
    A[k,j]:=B[k,j]
```

4.6.3 Semantics

A for clause causes the statement S which it precedes to be repeatedly executed zero or more times. In addition it performs a sequence of assignments to its controlled variable. The process may be visualized by means of the following picture:



In this picture the word initialize means: perform the first assignment of the for clause. Advance means: perform the next assignment of the for clause. Test determines if the last assignment has been done. If so the execution continues with the successor of the for statement. If not the statement following the for clause is executed.

4.6.4 The for list elements

The for list gives a rule for obtaining the values which are consecutively assigned to the controlled variable. This sequence of values is obtained from the for list elements by taking these one by one in the order in which they are written.

COMMENT: In order that the expansions given below are equivalent to the for statement, it would appear that the reference to the controlled variable can be evaluated more than once. This section, however, talks about the controlled variable implying that there is one unique evaluation. The reference to the controlled variable is thus evaluated only once.

```
(7)   iterate-for(cvar,for-elem,t,dn,vl) =

          cases: for-elem
          is-arithm-expr:
                 cases: eval-expr(for-elem,dn,vl)
                 (op¹,vl¹,Ω): let: v = convert(s-type(cvar),op¹)
                                   vl² = assign(cvar,v,vl¹)
                              int-st(t,dn,vl²)
                 (op¹,vl¹,lab¹): (vl¹,lab¹)
          is-step-until-elem:
               cases: eval-expr(s-init-expr(for-elem),dn,vl)
               (op¹,vl¹,Ω): let: v = convert(s-type(cvar),op¹)
                                 vl² = assign(cvar,v,vl¹)
                            iterate-step-until-elem(cvar,s-step-expr(for-elem),
                                                    s-until-expr(for-elem),t,dn,vl²)
               (op¹,vl¹,lab¹): (vl¹,lab¹)
          is-while-elem: iterate-while(cvar,for-elem,t,dn,vl)

      refs: is-arithm-expr 3.3, eval-expr 3, convert 4.2, assign 4.2, int-st 4
      note: Init expr eval once only.
      type: is-arithm-var X is-for-elem X is-st X is-dn X is-vl → is-vl X is-abn

(8)   iterate-step-until-elem(cvar,step-expr,until-expr,t,dn,vl) =

          cases: eval-until(cvar,step-expr,until-expr,dn,vl)
          (bool-op¹,vl¹,Ω):
               cases: s-value(bool-op¹)
               TRUE: (vl¹,Ω)
               FALSE:
                    cases: int-st(t,dn,vl¹)
                    (vl²,Ω):
                         cases: eval-step(cvar,step-expr,dn,vl²)
                         (op³,vl³,Ω):
                              let: v = convert(s-type(cvar),op³)
                                   vl⁴ = assign(cvar,v,vl³)
                              iterate-step-until-elem(cvar,step-expr,until-expr,t,dn,vl⁴)
                         (op³,vl³,lab³): (vl³,lab³)
                    (vl²,lab²): (vl²,lab²)
          (bool-op¹,vl¹,lab¹): (vl¹,lab¹)

      refs: int-st 4, convert 4.2, assign 4.2
      note: Step-expr is evaluated twice per iteration,
            until-expr is evaluated once per iteration.
      type: is-arithm-var X is-arithm-expr X is-arithm-expr X is-st X is-dn X is-vl →
                                                                  is-vl X is-abn
```


COMMENT: Since the for statement is taken to be equivalent to these expansions (except as mentioned above) the various expressions occurring within the for list elements can be evaluated more than once.

COMMENT: The controlled variable cannot be an activated function since the expansion of the for list elements prohibits this.

The sequence of values generated by each of the three species of for list elements and the corresponding execution of the statement S are given by the following rules:

4.6.4.1 Arithmetic expression. This element gives rise to one value, namely the value of the given arithmetic expression as calculated immediately before the corresponding execution of the statement S.

4.6.4.2 Step-until-element. An element of the form A step B until C, where A, B, and C are arithmetic expressions, gives rise to an execution which may be described most concisely in terms of additional ALGOL statements as follows:

```
    V:=A;
L1: if (V - C) * sign(B)>0 then goto Element exhausted;
    Statement S;
    V:=V + B;
    goto L1;
```

where V is the controlled variable of the for clause and Element exhausted points to the evaluation according to the next element in the for list, or if the step-until-element is the last of the list, to the next statement in the program.

```
(9)   eval-until(cvar,step-expr,until-expr,dn,vl) =

          eval-expr(μ0(<s-opr:GT>,
                       <s-op-1:μ0(<s-opr:MULT>,
                                  <s-op-1:μ0(<s-opr:MINUS>,
                                             <s-op-1:cvar>,
                                             <s-op-2:until-expr>)>,
                                  <s-op-2:μ0(<s-id:SIGN>,
                                             <s-type:INTG-PROC>,
                                             <s-arg-list:<step-expr>>)>)>,
                       <s-op-2:μ0(<s-type:INTG>,
                                  <s-value:0>)>)     ,dn,vl)

      refs: eval-expr 3
      type: is-arithm-var X is-arithm-expr X is-arithm-expr X is-dn X is-vl →
                  is-opt-bool-op X is-vl X is-abn


(10)  eval-step(cvar,step-expr,dn,vl) =

          eval-expr(μ0(<s-opr:PLUS>,
                       <s-op-1:cvar>,
                       <s-op-2:step-expr>),dn,vl)

      refs: eval-expr 3
      type: is-arithm-var X is-arithm-expr X is-dn X is-vl → is-opt-arithm-op X is-vl X is-abn


(11)  iterate-while(cvar,while-elem,t,dn,vl) =

          cases: eval-expr(s-init-expr(while-elem),dn,vl)
          (op¹,vl¹,Ω): let: v = convert(s-type(cvar),op¹)
                            vl² = assign(cvar,v,vl¹)
                       cases: eval-expr(s-while-expr(while-elem),dn,vl²)
                       (bool-op³,vl³,Ω):
                            cases: s-value(bool-op³)
                            FALSE: (vl³,Ω)
                            TRUE: cases: int-st(t,dn,vl³)
                                  (vl⁴,Ω): iterate-while(cvar,while-elem,t,dn,vl⁴)
                                  (vl⁴,lab⁴): (vl⁴,lab⁴)
                       (bool-op³,vl³,lab³): (vl³,lab³)
          (op¹,vl¹,lab¹): (vl¹,lab¹)

      refs: eval-expr 3, convert 4.2, assign 4.2, int-st 4
      type: is-arithm-var X is-while-elem X is-st X is-dn X is-vl → is-vl X is-abn
```


4.6.4.3 While-element. The execution governed by a for list element of the form E while F, where E is an arithmetic and F a Boolean expression, is most concisely described in terms of additional ALGOL statements as follows:

```
L3: V:=E;
    if ¬F then goto Element exhausted;
    Statement S;
    goto L3;
```

where the notation is the same as in 4.6.4.2 above.

4.6.5 The value of the controlled variable upon exit

Upon exit out of the statement S (supposed to be compound) through a goto statement the value of the controlled variable will be the same as it was immediately preceding the execution of the goto statement.

If the exit is due to exhaustion of the for list, on the other hand, the value of the controlled variable is undefined after the exit.

4.6.6 Go to leading into a for statement

The effect of a goto statement, outside a for statement, which refers to a label within the for statement, is undefined.

COMMENT: A goto statement resulting from the evaluation of an expression in the for list is considered external to the for statement, and thus is undefined if it refers to a label within the for statement.

Translation

(1) is-path(path) & is-proc-st•path(prog) & is-PROC•s-type•path(prog) ⊃
is-PROC•s-type•desc(s-id•path,prog) ∨ is-PROC•desc(s-id•path,prog)

(2) See also 3.2 for type procs in proc statements.

Abstract syntax

(3) is-proc-st = (<s-id:is-id>,
<s-act-par-list:is-act-par-list ∨ is-Ω>,
<s-type:is-type-proc ∨ is-PROC>)

refs: is-id 2.4, is-act-par 3.2, is-type-proc 5.4

Auxiliary Predicates

(4) is-changed-text =

This predicate is true of text satisfying is-text except that some identifiers may have been replaced by text satisfying is-act-par. Such text, obtained as a result of copying by name actual parameters, may not satisfy is-text.
(This leads to error in int-proc-body.)

Interpretation

(5) int-proc-st(t,dn,vl) =

cases: access(t,dn,vl)
(t¹,vl¹,Ω): cases: proc-access(t¹,dn,vl¹)
(op²,vl²,Ω): (vl²,Ω)
(op²,vl²,lab²): (vl²,lab²)
(t¹,vl¹,lab¹): (vl¹,lab¹)

refs: access 3, proc-access 3.2
type:is-proc-st X is-dn X is-vl → is-vl X is-abn

4.7.1 Syntax

<actual parameter> ::= <string> | <expression> | <array identifier> | <switch identifier> | <procedure identifier>

<letter string> ::= <letter> | <letter string> <letter>

<parameter delimeter> ::= , |) <letter string> : (

<actual parameter list> ::= <actual parameter> | <actual parameter list> <parameter delimiter> <actual parameter>

<actual parameter part> ::= <empty> | (<actual parameter list>)

<procedure statement> ::= <procedure identifier> <actual parameter part>

4.7.2 Examples

Spur(A)Order:(7)Result to:(V)
Transpose(W,v+1)
Absmax(A,N,M,Yy,I,K)
Innerproduct(A[t,P,u],B[P],10,P,Y)

These examples correspond to examples given in section 5.4.2.

4.7.3 Semantics

A procedure statement serves to invoke (call for) the execution of a procedure body (cf. section 5.4 Procedure declarations). Where the procedure body is a statement written in ALGOL the effect of this execution will be equivalent to the effect of performing the following operations on the program at the time of execution of the procedure statement:

```
(6)   non-type-proc(t,pr-set,dn,vl) =

          int-proc-body(change-text(t,pr-set),dn,vl)

      note: Called from activate-proc 3.2, which is called from proc-access 3.2.
      type: (is-block ∨ is-code) X is-pr-set X is-dn X is-vl → is-vl X is-abn


(7)   change-text(t,pr-set) =

          cases: t
          is-code: t
          is-id: cases: t ∈ firsts(pr-set)
                 TRUE: s(t,pr-set)
                 FALSE: t
          is-block: let: red-set¹ = del-set(pr-set,intr-ids(t))
                    μ0(<s-st-list:change-text(s-st-list(t),red-set¹)>,
                       <s-decl-pt:{change-text(d,red-set¹) | d∈s-decl-pt(t)}>)
          is-proc-desc: let: id-set² = {id | (∃i)(id = elem(i,s-form-par-list(t)))}
                        μ(t;<s-body:change-text(s-body(t),del-set(pr-set,id-set²))>)
          is-set: {change-text(el,pr-set) | el ∈ t}
          is-object: t
          is-ob: μ0({<sel:change-text(sel(t),pr-set)> | is-selector(sel) & ¬is-Ω(sel(t))})

      refs: is-code 4, is-id 2.4, is-block 4.1, intr-ids 4.1, is-proc-desc 5.4
      type: is-text X is-pr-set → is-changed-text


(8)   int-proc-body(t,dn,vl) =

          cases: t
          is-unlab-st: int-unlab-st(t,dn,vl)
          is-code: int-code(t,dn,vl)
          T: error

      refs: is-unlab-st 4, int-unlab-st 4, is-code 5.4
      error: If the result of change-block is not a well formed prog.
      type: is-changed-text X is-dn X is-vl → is-vl X is-abn


(9)   int-code(t,dn,vl) =

      This function is implementation defined.

      type: is-code X is-dn X is-vl → is-vl X is-abn
```


4.7.3.1 Value assignment (call by value). All formal parameters quoted in the value part of the procedure declaration heading are assigned the values (cf. section 2.8 Values and types) of the corresponding actual parameters, these assignments being considered as being performed explicitly before entering the procedure body.

> COMMENT: Thus the order in which primaries are referenced within expressions corresponding to by value parameters is arbitrary. (See 3)

The effect is as though an additional block embracing the procedure body were created in which these assignments were made to variables local to this fictitious block with types as given in the corresponding specifications (cf. section 5.4.5). As a consequence, variables called by value are to be considered as non-local to the body of the procedure, but local to the fictitious block (cf. section 5.4.3).

4.7.3.2 Name replacement (call by name). Any formal parameter not quoted in the value list is replaced, throughout the procedure body, by the corresponding actual parameter, after enclosing this latter in parentheses wherever syntactically possible. Possible conflicts between identifiers inserted through this process and other identifiers already present within the procedure body will be avoided by suitable systematic changes of the formal or local identifiers involved.

4.7.3.3 Body replacement and execution. Finally the procedure body, modified as above, is inserted in place of the procedure statement, and executed. If the procedure is called from a place outside the scope of any non-local quantity of the procedure body, the conflicts between the identifiers inserted through this process of body replacement and the identifiers whose declarations are valid at the place of the procedure statement or function designator will be avoided through suitable systematic changes of the latter identifiers.

4.7.4 Actual-formal correspondence

The correspondence between the actual parameters of the procedure statement and the formal parameters of the procedure heading is established as follows: The actual parameter list of the procedure statement must have the same number of entries as the formal parameter list of the procedure declaration heading. The correspondence is obtained by taking the entries of these two lists in the same order.

4.7.5 Restrictions

For a procedure statement to be defined it is evidently necessary that the operations on the procedure body defined in sections 4.7.3.1 and 4.7.3.2 lead to a correct ALGOL statement.

This poses the restriction on any procedure statement that the kind and type of each actual parameter be compatible with the kind and type of the corresponding formal parameter. Some important particular cases of this general rule are the following:

4.7.5.1 If a string is supplied as an actual parameter in a procedure statement or function designator, whose defining procedure body is an ALGOL 60 statement (as opposed to non-ALGOL code, cf. section 4.7.8), then this string can only be used within the procedure body as an actual parameter in further procedure calls. Ultimately it can only be used by a procedure body expressed in non-ALGOL code.

4.7.5.2 A formal parameter which occurs as a left part variable in an assignment statement within the procedure body and which is not called by value can only correspond to an actual parameter which is a variable (special case of expression).

4.7.5.3 A formal parameter which is used within the procedure body as an array identifier can only correspond to an actual parameter which is an array identifier of an array of the same dimensions. In addition, if the formal parameter is called by value, the local array created during the call will have the same subscript bounds as the actual array.

4.7.5.4 A formal parameter which is called by value cannot in general correspond to a switch identifier or a procedure identifier or a string, because these latter do not possess values (the exception is the procedure identifier of a procedure declaration which has an empty formal parameter part (cf. section 5.4.1) and which defines the value of a function designator (cf. section 5.4.4). This procedure identifier is in itself a complete expression).

> COMMENT: This exception is interpreted as though the function were called and its value passed to the corresponding formal parameter which is a variable. This is justified by noting that 4.7.4 talks of correspondence of actual-formal parameters. Thus here 'correspond to' is read as 'correspond to an actual parameter which is'.

4.7.5.5 Kind and type of actual parameters must be the same as those of the corresponding formal parameters, if called by name.

ECMA CHANGE

4.7.6 Deleted

4.7.7 Parameter delimiters

All parameter delimiters are understood to be equivalent. No correspondence between the parameter delimiters used in a procedure statement and those used in the procedure heading is expected beyond their number being the same. Thus the information conveyed by using the elaborate ones is entirely optional.

4.7.8 Procedure body expressed in code

The restrictions imposed on a procedure statement calling a procedure having its body expressed in non-ALGOL code evidently can only be derived from the characteristics of the code used and the intent of the user, and thus fall outside the scope of the reference language.

Translation

(1) The <declarations> in a <block head> are collected into a set.

(2) The <identifiers> introduced in a single <declaration> are collected into a set.

(3) The Translator rejects any <programs> in which errors of duplication would be hidden by (1) and (2) (e.g. real x,x; real y; real y;).

(4) The Translator rejects abstract programs in which an identifier is declared (or used as a label) more than once in the same scope:-

```
is-block(t) & decl-1 ∈ s-decl-pt(t) & decl-2 ∈ s-decl-pt(t) & decl-1 ≠ decl-2 ⊃
   disj(s-id-set(decl-1),s-id-set(decl-2)) &
   disj(s-id-set(decl-1),local-labs(t)) &
   (id-1 ∈ local-labs(t) & id-2 ∈ local-labs(t) & id-1 ≠id-2 ⊃
        make-st-sel(id-1,t) ≠ make-st-sel(id-2,t))
```

refs: is-block 4.1, local-labs 4.1, make-st-sel 4

Abstract syntax

(5) is-desc = is-var-desc ∨ is-array-desc ∨ is-switch-desc ∨ is-proc-desc

refs: is-var-desc 5.1, is-array-desc 5.2, is-switch-desc 5.3, is-proc-desc 5.4

(6) is-decl = (<s-id-set:is-id-set>,
<s-desc:is-desc>)

refs: is-id 2.4
note: The s-id-set of switch or procedure declarations will always be a unit set.

Auxiliary predicates

(7) is-label-desc = (<s-type:is-LABEL>)

Declarations serve to define certain properties of the quantities used in the program, and to associate them with identifiers. A declaration of an identifier is valid for one block. Outside this block the particular identifier may be used for other purposes (cf. section 4.1.3).

Dynamically this implies the following: at the time of an entry into a block (through the begin, since the labels inside are local and therefore inaccessible from outside) all identifiers declared for the block assume the significance implied by the nature of the declarations given. If these identifiers had already been defined by other declarations outside they are for the time being given a new significance. Identifiers which are not declared for the block, on the other hand, retain their old meaning.

At the time of an exit from a block (through end, or by a go to statement) all identifiers which are declared for the block lose their local significance.

ECMA CHANGE

Apart from labels and formal parameters of procedure declarations and with the possible exception of those for standard functions (cf. sections 3.2.4 and 3.2.5) all identifiers of a program must be declared. No identifier may be declared more than once in any one block head.

Syntax

<declaration> ::= <type delaration> | <array declaration> | <switch declaration> |
<procedure declaration>

Interpretation

(8) augment-dn(decl-set,dn) =

mod-set(dn,{<id,make-den(desc)>|(∃decl)(decl∈decl-set & desc=s-desc(decl) & id∈s-id-set(decl))})

type: is-intr-set X is-dn → is-dn

(9) make-den(desc) =

cases: desc
is-var-desc: μ_0(<s-type:desc>)
is-array-desc: desc
is-switch-desc: μ_0(<s-switch-list:desc>,<s-type:SWITCH>)
is-label-desc: desc
is-proc-desc: desc

refs: is-var-desc 5.1, is-array-desc 5.2, is-switch-desc 5.3, is-proc-desc 5.4
type: (is-desc ∨ is-label-desc) → is-den

5.1 TYPE DECLARATIONS

Abstract syntax

(1) is-arithm = is-INTG ∨ is-REAL

(2) is-type = is-arithm ∨ is-BOOL

(3) is-var-desc = is-type

5.1.1 Syntax

<type list> ::= <simple variable> | <simple variable> , <type list>

<type> ::= real | integer | Boolean

<type declaration> ::= <type> <type list>

ECMA CHANGE

5.1.2 Examples

```
integer p,q,s
own Boolean Acryl,n
```

NON ECMA LANGUAGE

Interpretation

(4) see make-den :- is-var-desc

refs: make-den 5

5.2 ARRAY DECLARATIONS

Translation

(1) The Translator rejects any program in which array bound expressions use local names:-

is-block(block) ⊃ (∀decl)(decl ∈ s-decl-pt(block) & is-array-desc(s-desc(decl)) ⊃
(sel•s-bounds(s-desc(decl)) = id-1 ⊃ ¬(id-1 ∈ intr-ids(block))))

refs: is-block 4, intr-ids 4.1
note: Declarations of the form array are treated as real array (see A.R. 5.2.3.3).

Abstract syntax

(2) is-arithm-array = is-REAL-ARRAY ∨ is-INTG-ARRAY

(3) is-type-array = is-arithm-array ∨ is-BOOL-ARRAY

(4) is-bound-pair = (<s-lbd:is-arithm-expr>,
<s-ubd:is-arithm-expr>)

refs: is-arithm-expr 3.3

(5) is-array-desc = (<s-type:is-type-array>,
<s-bounds:(is-bound-pair-list & ¬is-<>)>)

5.1.3 Semantics

Type declarations serve to declare certain identifiers to represent simple variables of a given type. Real declared variables may only assume positive or negative values including zero. Integer declared variables may only assume positive and negative integral values, including zero. Boolean declared variables may only assume the values true and false.

In arithmetic expressions any position which can be occupied by a real declared variable may be occupied by an integer declared variable.

ECMA CHANGE

5.2 ARRAY DECLARATIONS

5.2.1 Syntax

<lower bound> ::= <arithmetic expression>

<upper bound> ::= <arithmetic expression>

<bound pair> ::= <lower bound> : <upper bound>

Auxiliary predicates

(6) is-array-decl = (<s-id-set:is-id-set>,
<s-desc:is-array-desc>)

refs: is-id 2.4

Interpretation

(7) see make-den :- is-array-desc

refs: make-den 5

(8) eval-array-decls(decl-set,dn,vl) =

cases: decl-set
(∀decl¹)(decl¹ ∈ decl-set & is-array-decl(decl¹) ⊃ is-array-den(s-desc(decl¹))):
(decl-set,vl,Ω)
T: for some decl¹ ∈ decl-set & is-array-decl(decl¹) & ¬is-array-den(s-desc(decl¹))
cases: eval-array-bds(s-desc(decl¹),dn,vl)
(desc²,vl²,Ω): cases: eval-array-decls(decl-set - {decl¹},dn,vl²)
(decl-set³,vl³,Ω): (decl-set³ ∪ {μ(decl²;<s-desc:desc²>)},vl³,Ω)
(decl-set³,vl³,lab³): (Ω,vl³,lab³)
(desc²,vl²,lab²): (Ω,vl²,lab²)

type: is-decl-set X is-dn X is-vl → is-opt-decl-set X is-vl X is-abn

<bound pair list> ::= <bound pair> | <bound pair list> , <bound pair>

<array segment> ::= <array identifier> [<bound pair list>] | <array identifier> , <array segment>

<array list> ::= <array segment> | <array list> , <array segment>

<array declaration> ::= array <array list> | <type> array <array list>

ECMA CHANGE

5.2.2 Examples

```
array a,b,c[7:n,2:m],s[-2:10]
own integer array A[if c<0 then 2 else 1:20]
        NON ECMA LANGUAGE
real array q[-7:-1]
```

5.2.3 Semantics

An array declaration declares one or several identifiers to represent multidimensional arrays of subscripted variables and gives the dimensions of the arrays, the bounds of the subscripts and the types of the variables.

5.2.3.1 Subscript bounds. The subscript bounds for any array are given in the first subscript bracket following the identifier of this array in the form of a bound pair list. Each item of this list gives the lower and upper bound of a subscript in the form of two arithmetic expressions separated by the delimiter : . The bound pair list gives the bounds of all subscripts taken in order from left to right.

5.2.3.2 Dimensions. The dimensions are given as the number of entries in the bound pair lists.

5.2.3.3 Types. All arrays declared in one declaration are of the same quoted type. If no type declarator is given the type real is understood.

5.2.4 Lower upper bound expressions

5.2.4.1 The expressions will be evaluated in the same way as subscript expressions (cf. section 3.1.4.2).

COMMENT: References within bound pair lists are performed in arbitrary order; bound pair lists are evaluated in arbitrary order within a declaration part. However, references of different bound pair lists are not intermixed.

```
(9)   eval-array-bds(desc,dn,vl) =

        cases: access(s-bounds(desc),dn,vl)
        (abds,vl¹,Ω):
            let: ebds = μ0({<sel-1•elem(i):apply(sel-1•elem(i)(abds))> |
                                (sel-1 = s-lbd ∨ sel-1 = s-ubd) & 1≤i≤length(abds)})
            cases: ebds
            (∀i)(1≤i≤length(ebds) ⊃ s-lbd(elem(i,ebds)) ≤ s-ubd(elem(i,ebds))):
                (μ(desc;<s-bounds:ebds>),vl¹,Ω)
            T: error
        (abds,vl¹,lab¹): (Ω,vl¹,lab¹)

    refs: access 3, apply 3
    error: No array is defined if any upper bound is less than the corresponding lower bound.
    type: is-array-desc X is-dn X is-vl → is-opt-array-den X is-vl X is-abn
```

5.3 SWITCH DECLARATIONS

Abstract syntax

```
(1)   is-switch-desc = (is-des-expr-list & ¬is-<>)

        refs: is-des-expr 3.5
```


5.2.4.2 The expressions can only depend on variables and procedures which are non-local to the block for which the array declaration is valid.

COMMENT: It is assumed that "variables" can be taken to include labels at this point. Thus a function reference within such expressions may not have any local labels as actual parameters.

Consequently in the outermost block of a program only array declarations with constant bounds may be declared.

5.2.4.3 An array is defined only when the values of all upper subscript bounds are not smaller than those of the corresponding lower bounds.

5.2.4.4 The expressions will be evaluated once at each entrance into the block.

5.2.5 The identity of subscripted variables

The identity of a subscripted variable is not related to the subscript bounds given in the array declaration . However , the values of the corresponding subscripted variables will, at any time, be defined only for those of these variables which have subscripts within the most recently calculated subscript bounds.

ECMA CHANGE

5.3 SWITCH DECLARATIONS

5.3.1 Syntax

<switch list> ::= <designational expression> | <switch list> , <designational expression>

<switch declaration> ::= switch <switch identifier> := <switch list>

5.3.2 Examples

```
switch S:=S1, S2, Q[m], if v>-5 then S3 else S4
switch Q:=p1, w
```

Interpretation

(2) see make-den :- is-switch-desc

refs: make-den 5

5.4 PROCEDURE DECLARATIONS

Translation

(1) The entries in the <specification part> are collected into a set which associates one copy of the appropriate <specifier> with each <identifier>.

(2) The <identifiers> in the <value part> are collected into a set.

(3) The Translator rejects any <program> in which errors of duplication would be hidden by (1) and (2) (e.g. procedure p(x); value x,x; real x; real x; ...).

(4) The <specifier> array is treated as real array.

5.3.3 Semantics

A switch declaration defines the set of values of the corresponding switch designators. These values are given one by one as the values of the designational expressions entered in the switch list. With each of these designational expressions there is associated a positive integer, 1,2 ..., obtained by counting the items in the list from left to right. The value of the switch designator corresponding to a given value of the subscript expression (cf. section 3.5 Designational expressions) is the value of the designational expression in the switch list having this given value as its associated integer.

5.3.4 Evaluation of expressions in the switch list

An expression in the switch list will be evaluated every time the item of the list in which the expression occurs is referred to, using the current values of all variables involved.

5.3.5 Influence of scopes

If a switch designator occurs outside the scope of a quantity entering into a designational expression in the switch list, and an evaluation of this switch designator selects the designational expression, then the conflicts between the identifiers for the quantities in this expression and the identifiers whose declarations are valid at the place of the switch designator will be avoided through suitable systematic changes of the latter identifiers.

5.4 PROCEDURE DECLARATIONS

(5) The body of a procedure (except the case of code) is formed into a block (see A.R. 5.4.3).

(6) The Translator rejects abstract programs in which an identifier appears in more than one spec:-

```
is-proc-desc(pd) & spec-1 ∈ s-spec-pt(pd) & spec-2 ∈ s-spec-pt(pd) ⊃
   (s-id(spec-1) = s-id(spec-2) ⊃ spec-1 = spec-2)
```

(7) The Translator rejects abstract programs in which the same identifier occurs in more than one position in the form-par-list:-

```
is-proc-desc(pd) & elem(i,s-form-par-list(pd)) = elem(j,s-form-par-list(pd)) ⊃ i=j
```

(8) The Translator rejects any programs in which a formal parameter does not have a corresponding specifier (see A.R. 5.4.5) :-

```
is-proc-desc(pd) & 1≤i≤length•s-form-par-list(pd) ⊃
   (∃spec)(spec∈s-spec-pt(pd) & s-id(spec) = elem(i)•s-form-par-list(pd))
```

(9) The Translator rejects any programs in which an identifier appears in the value part but not in the formal parameter list :-

```
is-proc-desc(pd) & id∈s-value-pt(pd) ⊃ (∃i)(elem(i)•s-form-par-list = id)
```

(10) The Translator rejects abstract programs in which procedure, string or switch parameters appear in the value-pt:-

```
is-proc-desc(pd) & spec ∈ s-spec-pt(pd) &
   (is-type-proc(s-specifier(spec)) ∨
    is-PROC(s-specifier(spec)) ∨
    is-STRING(s-specifier(spec)) ∨
    is-SWITCH(s-specifier(spec))) ⊃ ¬(s-id(spec) ∈ s-value-pt(pd))
```

(11) The following function is used to determine the type of references etc. :-

```
desc-proc(id,path-el•path,t) =

        cases: id
        (∃spec)(spec ∈ s-spec-pt•path-el•path(t) & id = s-id(spec)):
             let: spec¹ = (ι spec)(spec∈s-spec-pt•path-el•path(t) & id=s-id(spec))
             s-specifier(spec¹)
        T: desc-1(id,path,t)

    type: is-id X is-path X is-program → (is-specifier ∨ is-desc ∨ is-label-desc)
```

Abstract syntax

(12) is-code =

This predicate is implementation defined. The objects satisfying it are distinct elementary objects (thus is-sel(sel) & is-code(t) ⊃ is-Ω•sel(t)).

(13) is-type-proc = is-REAL-PROC ∨ is-INTG-PROC ∨ is-BOOL-PROC

(14) is-specifier = is-type ∨ is-type-array ∨ is-type-proc ∨ is-PROC ∨ is-LABEL ∨ is-STRING ∨ is-SWITCH

refs: is-type 5.1, is-type-array 5.2

(15) is-spec = (<s-id:is-id>, <s-specifier:is-specifier>)

refs: is-id 2.4

(16) is-proc-desc = (<s-type:is-type-proc ∨ is-PROC>, <s-form-par-list:is-id-list>, <s-spec-pt:is-spec-set>, <s-value-pt:is-id-set>, <s-body:is-block ∨ is-code>)

refs: is-id 2.4, is-block 4
note: The body of a procedure, unless code, is made into a block.

5.4.1 Syntax

<formal parameter> ::= <identifier>

<formal parameter list> ::= <formal parameter> | <formal parameter list> <parameter delimeter> <formal parameter>

<formal parameter part> ::= <empty> | (<formal parameter list>)

<identifier list> ::= <identifier> | <identifier list> , <identifier>

<value part> ::= value <identifier list> ; | <empty>

<specifier> ::= string | <type> | array | <type> array | label | switch | procedure | <type> procedure

<specification part> ::= <empty> | <specifier> <identifier list> ; | <specification part> <specifier> <identifier list> ;

<procedure heading> ::= <procedure identifier> <formal parameter part> ; <value part> <specification part>

<procedure body> ::= <statement> | <code>

<procedure declaration> ::= procedure <procedure heading> <procedure body> | <type> procedure <procedure heading> <procedure body>

5.4.2 Examples (see also the examples at the end of the report)

```
procedure Spur(a)Order:(n)Result:(s); value n;
array a; integer n; real s;
begin integer k;
s:=0;
for k:=1 step 1 until n do s:=s+a[k,k]
end
```

Interpretation

(17) see mk-den :- is-proc-desc

refs: make-den 5


```
procedure Transpose(a) Order:(n) ; value n;
array a; integer n;
begin real w; integer i,k;
for i:=1 step 1 until n do
     for k:=1+i step 1 until n do
     begin w:=a[i,k];
           a[i,k]:=a[k,i];
           a[k,i]:=w
     end
end Transpose

integer procedure Step(u) ; real u;
Step:= if 0≤u & u≤1 then 1 else 0

procedure Absmax(a) size:(n,m) Result:(y) Subscripts:(i,k) ;
comment The absolute greatest element of the matrix a,of
     size n by m is transferred to y, and the subscripts
     of this element to i and k;
array a; integer n,m,i,k; real y;
begin integer p,q;
y:=0;
for p:=1 step 1 until n do for q:=1 step 1 until m do
if abs(a[p,q])>y then begin y:=abs(a[p,q]) ;

i:=p; k:=q end end Absmax

procedure Innerproduct(a,b) Order:(k,p) Result:(y) ;
value k;
integer k,p; real y,a,b;
begin real s; s:=0;
for p:=1 step 1 until k do s:=s+a*b;
y:=s;
end Innerproduct
```

5.4.3 Semantics

A procedure declaration serves to define the procedure associated with a procedure identifier. The principal constituent of a procedure declaration is a statement or a piece of code, the procedure body, which through the use of procedure statements and/or function designators may be activated from other parts of the block in the head of which the procedure declaration appears. Associated with the body is a heading, which specifies certain identifiers occurring within the body to represent formal parameters. Formal parameters in the procedure body will, whenever the procedure is activated (cf. section 3.2 Function designators and section

4.7 Procedure statements) be assigned the values of or replaced by actual parameters. Identifiers in the procedure body which are not formal will be either local or non-local to the body depending on whether they are declared within the body or not. Those of them which are non-local to the body may well be local to the block in the head of which the procedure declaration appears. The procedure body always acts like a block, whether it has the form of one or not. Consequently the scope of any label labelling a statement within the body or the body itself can never extend beyond the procedure body. In addition, if the identifier of a formal parameter is declared anew within the procedure body (including the case of its use as a label as in section 4.1.3), it is thereby given a local significance and actual parameters which correspond to it are inaccessible throughout the scope of this inner local quantity.

5.4.4 Values of function designators

For a procedure declaration to define the value of a function designator there must, within the procedure body, occur one or more explicit assignment statements with the procedure identifier in a left part; at least one of these must be executed, and the type associated with the procedure identifier must be declared through the appearance of a type declarator as the very first symbol of the procedure declaration. The last value so assigned is used to continue the evaluation of the expression in which the function designator occurs. Any occurrence of the procedure identifier within the body of the procedure other than in a left part in an assignment statement denotes activation of the procedure.

COMMENT: If the procedure is terminated by a goto out of the procedure body, the value of the procedure is not used.

5.4.5 Specifications

In the heading a specification part, giving information about the kinds and types of the formal parameters by means of an obvious notation, may be included. In this part no formal parameter may occur more than once. Specifications of all formal parameters if any must be supplied.

ECMA CHANGE

5.4.6 Code as procedure body

It is understood that the procedure body may be expressed in non-ALGOL language. Since it is intended that the use of this feature should be

entirely a question of hardware representation, no further rules concerning this code language can be given within the reference language.

Examples of procedure declarations.
NON ECMA LANGUAGE USED

Example 1

```
procedure euler(fct,sum,eps,tim);value eps,tim;
integer tim;
real procedure fct; real sum,eps;
comment euler computes the sum of fct(i) for i from zero
up to infinity by means of a suitably refined euler
transformation. The summation is stopped as soon as tim
times in succession the absolute value of the terms of
the transformed series are found to be less than eps.
Hence, one should provide a function fct with one integer
argument, an upper bound eps, and an integer tim. The
output is the sum sum. euler is particularly efficient in
the case of a slowly convergent or divergent alternating
series;
begin integer i,k,n,t; array m[0:15]; real mn,mp,ds;
i:=n:=t:=0; m[0]:=fct(0); sum:=m[0]/2;
nextterm: i:=i+1; mn:=fct(i);
     for k:=0 step 1 until n do
          begin mp:=(mn+m[k])/2; m[k]:=mn;
          mn:=mp end means;
     if (abs(mn)<abs(m[n])) & (n<15) then
          begin ds:=mn/2; n:=n+1;
          m[n]:=mn end accept
     else ds:=mn;
     sum:=sum+ds;
     if abs(ds)<eps then t:=t+1 else t:=0;
     if t<tim then goto nextterm
end euler
```

Example 2

```
procedure RK(x,y,n,FKT,eps,eta,xE,yE,fi);
     value x,y; integer n;
Boolean fi; real x,eps,eta,xE; array y,yE;
     procedure FKT;
```

```
comment: RK integrates the system
     y[k](x) = f(k)(x,y[1],y[2],...,yb)    (k=1,2,...,n)
of differential equations with the method of Runge-Kutta
with automatic search for appropriate length of integration
step. Parameters are: The initial values x and y[k] for
x and the unknown functions y(k)(x). The order n of the
system. The procedure FKT(x,y,n,z) which represents the
system to be integrated, i.e. the set of functions f(k).
The tolerance values eps and eta which govern the
accuracy of the numerical integration. The end of the
integration interval xE. The output parameter yE which
represents the solution at x=xE. The Boolean variable fi,
which must always be given the value true for an isolated
or first entry into RK. If,however, the functions y must
be available at several meshpoints x(0),x(1),...,x(n),
then the procedure must be called repeatedly (with x=x(k)
xE=x(k+1), for k=0,1,...,n-1) and then the later calls
may occur with fi=false which saves computing time. The
input parameters of FKT must be x,y,n, the output
parameter z represents the set of derivatives
z[k]=f(k)(x,y[1],y[2],...,y[n]) for x and the actual y's.
A procedure comp enters as a non-local identifier;
begin
     array z,y1,y2,y3[1:n]; real x1,x2,x3,H;
          Boolean out;
     integer k,j; own real s,Hs;
     procedure RK1ST(x,y,h,xe,ye); real x,h,xe;
          array y,ye;
     comment : RK1ST integrates one single RUNGE-KUTTA
     step with initial values x,y[k] which yields the
     output parameters xe=x+h and ye[k], the latter being
     the solution at xe. IMPORTANT: the parameters n,FKT,
     z enter RK1ST as non-local entities;
     begin
          array w[1:n], a[1:5]; integer k,j;
          a[1]:=a[2]:=a[5]:=h/2; a[3]:=a[4]:=h;
          xe:=x;
          for k:=1 step 1 until n do ye[k]:=w[k]:=y[k];
          for j:=1 step 1 until 4 do
          begin
               FKT(xe,w,n,z);
               xe:=x+a[j];
               for k:=1 step 1 until n do
```

```
                begin
                    w[k]:=y[k]+a[j]*z[k];
                    ye[k]:=ye[k]+a[j+1]*z[k]/3
                end k
            end j
    end RK1ST;
BEGIN OF PROGRAM:
    if fi then begin H:=xE-x; s:=0 end
    else H:=Hs; out:=false;
AA: if (x+2.01*H-xE>0) ≡ (H>0) then
    begin Hs:=H; out:=true;H:=(xE-x)/2
    end if;
    RK1ST(x,y,2*H,x1,y1);
BB: RK1ST(x,y,H,x2,y2);RK1ST(x2,y2,H,x3,y3);
    for k:=1 step 1 until n do
        if comp(y1[k],y3[k],eta)>eps then goto CC;
    comment : comp(a,b,c) is a function designator the
    value of which is the absolute value of the
    difference of the mantissae of a and b, after the
    exponents of these quantities have been made equal
    to the largest of the exponents of the originally
    given parameters a,b,c;
    x:=x3; if out then goto DD;
    for k:=1 step 1 until n do y[k]:=y3[k];
    if s=5 then begin s:=0;H:=2*H end if;
    s:=s+1; goto AA;
CC: H:=0.5*H; out:=false ; x1:=x2;
    for k:=1 step 1 until n do y1[k]:=y2[k];
    goto BB;
DD: for k:=1 step 1 until n do yE[k]:=y3[k]
    end RK
```

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APPENDIX: Cross Reference Index

NAME	DCLN	USED IN
change-text	4.7 (7)	4.1 (14) , 4.7 (6) , 4.7 (7)
construct-pairs	4.1 (12)	4.1 (11) , 4.1 (12)
convert	4.2 (21)	3 (6) , 3.1 (23) , 3.2 (18) , 4.2 (20) , 4.6 (7) , 4.6 (8) , 4.6 (11)
convert-array	3.2 (19)	3.2 (18)
convert-array-el	3.2 (20)	3.2 (19)
convert-one-sub	3.1 (23)	3.1 (22)
convert-subs	3.1 (22)	3.1 (21) , 4.2 (18)
cue-int-st	4 (9)	4 (7) , 4 (9) , 4 (10) , 4.1 (17)
cue-int-st-list	4.1 (17)	4 (10) , 4.1 (16) , 4.1 (17)
cue-int-unlab-st	4 (10)	4 (9)
del-set	1 (12)	4.1 (18) , 4.6 (6) , 4.7 (7)
desc	1 (51)	
desc-block	4.1 (5)	1 (52)
desc-proc	5.4 (11)	1 (52)
desc-1	1 (52)	1 (51) , 1 (52) , 4.1 (5) , 5.4 (11)
disj	1 (59)	
entier	1 (49)	3.2 (20) , 3.3 (38) , 4.2 (21)
epilogue	4.1 (18)	3.2 (16) , 3.2 (23) , 4.1 (10)
eval-act-par	3.2 (18)	3.2 (17)
eval-act-par-list	3.2 (17)	3.2 (16)
eval-array-bds	5.2 (9)	5.2 (8)
eval-array-decls	5.2 (8)	4.1 (10) , 5.2 (8)

NAME	DCLN	USED IN
eval-expr	3 (4)	4.2 (16) , 4.2 (19) , 4.3 (2) , 4.5 (3) , 4.6 (7) , 4.6 (9) , 4.6 (10) 4.6 (11)
eval-lp	4.2 (18)	4.2 (17) , 4.6 (5)
eval-lp-list	4.2 (17)	4.2 (16) , 4.2 (17)
eval-step	4.6 (10)	4.6 (8)
eval-subs	4.2 (19)	4.2 (18) , 4.2 (19)
eval-until	4.6 (9)	4.6 (8)
firsts	1 (14)	4.7 (7)
fn-access	3.2 (14)	3 (4) , 3 (8)
head	1 (43)	4.2 (17) , 4.2 (19) , 4.2 (22) , 4.6 (6)
insert-ret	3.2 (24)	3.2 (23) , 3.2 (24)
int-assign-st	4.2 (16)	4 (8)
int-block	4.1 (10)	4 (8) , 4.1 (9)
int-block-body	4.1 (15)	4.1 (10)
int-code	4.7 (9)	4.7 (8)
int-cond-st	4.5 (3)	4 (8)
int-for-st	4.6 (5)	4 (8)
int-goto-st	4.3 (2)	4 (8)
int-proc-body	4.7 (8)	4.7 (6)
int-proc-st	4.7 (5)	4 (8)
int-program	4.1 (9)	
int-st	4 (7)	4 (9) , 4.1 (16) , 4.5 (3) , 4.6 (7) , 4.6 (8) , 4.6 (11)
int-st-list	4.1 (16)	4 (8) , 4.1 (15) , 4.1 (16) , 4.1 (17)

NAME	DCLN	USED IN
is-intg-lp	4.2(7)	4.2(11),4.2(15),4.2(20)
is-intg-op	1(35)	1(37),3.3(22)
is-intg-prefix-expr	3.3(4)	3.3(22),3.3(24)
is-intg-simple-var	3.1(9)	3.1(14),3.1(17)
is-intg-subscr-var	3.1(10)	3.1(14),3.1(18)
is-intg-val	2.5(2)	1(1),1(17),1(24),1(26),1(27),1(35),1(49),2.5(3),2.5(5) 3.1(24)
is-intg-var	3.1(14)	3.1(15),3.3(22),4.2(7)
is-intr	1(54)	
is-lab-sel	1(56)	
is-lab-selector	1(55)	1(56)
is-label-const	3.5(7)	2(4)
is-label-decl	1(57)	1(54)
is-label-den	1(19)	1(22)
is-label-desc	5(7)	1(57),5(9)
is-label-op	1(39)	1(40),3.2(16),3.2(18),3.2(22),3.5(8)
is-label-var	3.5(6)	3.1(17),3.5(8)
is-list	1(42)	1(47)
is-lp	4.2(15)	
is-non-neg-intg-const	2.5(6)	3.3(9),3.3(14),3.3(33)
is-non-neg-intg-val	2.5(3)	2.5(6)
is-non-type-proc-name	3.2(6)	3.2(8)
is-ob	1(2)	1(9),3(6),3.2(24),4.7(7)

NAME	DCLN	USED IN
is-object	1(1)	3(6),3(7),3.2(24),4.7(7)
is-op	1(40)	3(3),3(6),3(7),3(9)
is-op-expr	3(3)	3(3),3(6)
is-opt-(pred)	1(62)	
is-path	1(6)	1(58)
is-path-el	1(5)	1(6)
is-pr	1(9)	
is-proc-den	1(21)	1(22),4.1(11)
is-proc-desc	5.4(16)	1(52),4.1(13),4.7(7),5(5),5(9)
is-proc-st	4.7(3)	3(5),4(5),4(8)
is-program	4.1(8)	1(58)
is-real-activated-fn	4.2(4)	4.2(5),4.2(14)
is-real-assign-st	4.2(10)	4.2(13)
is-real-cond-expr	3.3(19)	3.3(21),3.3(26)
is-real-cond-expr-1	3.3(16)	3.3(19)
is-real-cond-expr-2	3.3(17)	3.3(19)
is-real-cond-expr-3	3.3(18)	3.3(19)
is-real-const	2.5(4)	2.5(7)
is-real-expr	3.3(21)	3.2(21),3.3(3),3.3(6),3.3(7),3.3(8),3.3(16),3.3(17) 3.3(18),3.3(23),3.3(29)
is-real-funct-ref	3.2(9)	3.2(12),3.3(21)
is-real-infix-expr	3.3(11)	3.3(21),3.3(25)
is-real-infix-expr-1	3.3(6)	3.3(11)

NAME	DCLN	USED IN
is-real-infix-expr-2	3.3(7)	3.3(11)
is-real-infix-expr-3	3.3(8)	3.3(11)
is-real-infix-expr-4	3.3(9)	3.3(11)
is-real-infix-expr-5	3.3(10)	3.3(11)
is-real-infix-opr	3.3(5)	3.3(6),3.3(7),3.3(8),3.3(28),3.3(32)
is-real-lp	4.2(5)	4.2(10),4.2(15),4.2(20)
is-real-op	1(34)	1(37),3.3(21)
is-real-prefix-expr	3.3(3)	3.3(21),3.3(24)
is-real-simple-var	3.1(7)	3.1(13),3.1(17)
is-real-subscr-var	3.1(8)	3.1(13),3.1(18)
is-real-val	2.5(1)	1(1),1(24),1(34),2.5(4)
is-real-var	3.1(13)	3.1(15),3.3(21),4.2(5)
is-sel	1(4)	
is-selector	1(3)	1(5),1(60),3(6),3(7),3.2(24),4.7(7)
is-set	1(7)	1(1),1(5),1(8),3.2(24),4.7(7)
is-simple-val	1(25)	1(29),3.2(13)
is-simple-var	3.1(17)	3(6),3(7),3(8),3.1(19),4.2(18),4.2(23),4.6(6)
is-spec	5.4(15)	1(21),5.4(16)
is-specifier	5.4(14)	5.4(15)
is-st	4(6)	4.1(6),4.1(7),4.5(2),4.6(4)
is-step-until-elem	4.6(2)	4.6(3),4.6(7)
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is-subscr-var	3.1(18)	3(7),3(8),3.1(19),4.2(18),4.2(23),4.6(6)
is-switch-den	1(20)	1(22)
is-switch-des	3.5(4)	3(3),3(6),3.5(8)
is-switch-desc	5.3(1)	5(5),5(9)
is-text	1(58)	
is-type	5.1(2)	1(16),3.1(20),3.2(13),5.1(3),5.4(14)
is-type-array	5.2(3)	1(18),3.2(7),3.2(21),5.2(5),5.4(14)
is-type-const	2(3)	2(4)
is-type-den	1(16)	1(22)
is-type-op	1(38)	1(40),3.2(22)
is-type-proc	5.4(13)	1(21),3.2(16),3.2(21),4.7(3),5.4(14),5.4(16)
is-unlab-st	4(5)	4(6),4.7(8)
is-val	1(29)	1(30)
is-value-parm	3(10)	3(6),3(7),3.2(15),3.2(16)
is-var	3.1(19)	3(3)
is-var-desc	5.1(3)	5(5),5(9)
is-vl	1(30)	
is-while-elem	4.6(1)	4.6(3),4.6(7)
iterate-for	4.6(7)	4.6(6)
iterate-for-list	4.6(6)	4.6(5),4.6(6)
iterate-step-until-elem	4.6(8)	4.6(7),4.6(8)

NAME	DCLN	USED IN
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local-labs	4.1(4)	4.1(3),4.1(5),4.1(10)
main-pt	1(60)	1(61),4(10),4.1(16),4.1(17)
make-den	5(9)	5(8)
make-st-sel	4(4)	4(7),4(9),4.1(16),4.1(17)
match	3.2(21)	3.2(18)
mk-op	1(41)	3.1(20),3.1(21),3.2(18),3.3(29),3.3(30),3.3(32),3.3(33) 3.3(34),3.4(9),3.4(10),3.4(12)
mk-pairs	4.1(11)	3.2(16),3.2(23),4.1(10)
mod-set	1(13)	3.2(16),3.2(23),4.1(10),4.2(23),4.6(6),5(8)
non-type-proc	4.7(6)	3.2(16),3.2(23)
one-access	3(8)	3(5)
proc-access	3.2(15)	3.2(14),4.7(5)
ready-set	3(7)	3(5),3(7)
real-arithm-infix-value	3.3(37)	3.3(32),3.3(36)
real-power-value	3.3(36)	3.3(32)
real-prefix-value	3.3(31)	3.3(30)
reduce-cond-switch	3(6)	3(5),3(6)
rest-pt	1(61)	4(9),4(10),4.1(16),4.1(17)
s	1(11)	3(6),3(7),3.1(20),3.1(21),3.1(24),3.2(15),3.2(23),4.2(18) 4.2(23),4.6(6),4.7(7)

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sign	1(48)	1(50),3.3(38)
simp-var-access	3.1(20)	3(8)
subscr-var-access	3.1(21)	3(8)
tail	1(44)	4.2(17),4.2(19),4.2(22),4.6(6)
type-proc	3.2(23)	3.2(16)
val-den	3.2(22)	3.2(16)